3.4: Graphing using Slope (cont'd)

Note Title

9/30/2015 ~~2/5/2015~~

Ex: Find Calculate the slope of the line passing through the points $(-4, -1)$ and $(3, -6)$.

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-(-1)}{3-(-4)} = \frac{-6+1}{3+4} = \frac{-5}{7} = \boxed{-\frac{5}{7}}$

Note: We get the same answer if we subtract the points in the opposite order.

Let $(x_1, y_1) = (3, -6)$
$(x_2, y_2) = (-4, -1)$

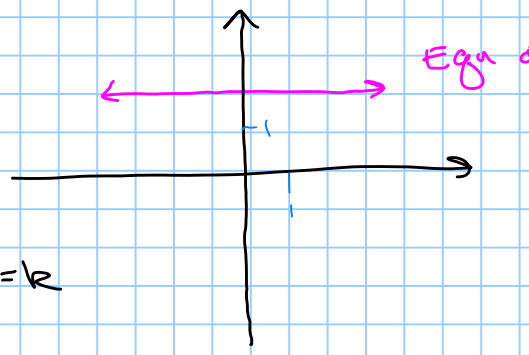
$m = \frac{-1-(-6)}{-4-3} = \frac{-1+6}{-7} = \frac{5}{-7} = \boxed{-\frac{5}{7}}$

Recall

Horizontal Lines:

Slope is 0

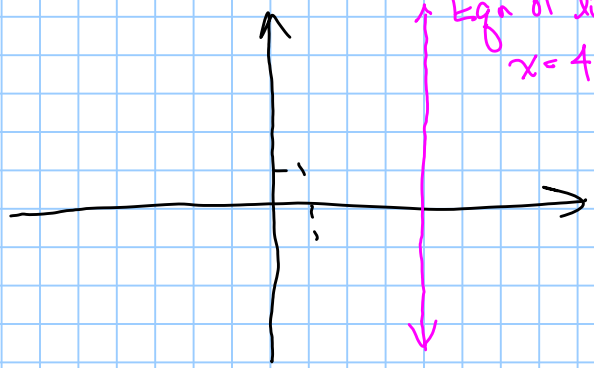
Equation has the form $y = k$ (k a constant)



Vertical lines:

Slope is undefined.

Equation has the form $x = k$ (k a constant)



Recall: Slope-intercept form of a line:

$y = mx + b$, where m is the slope and b is the y-intercept.

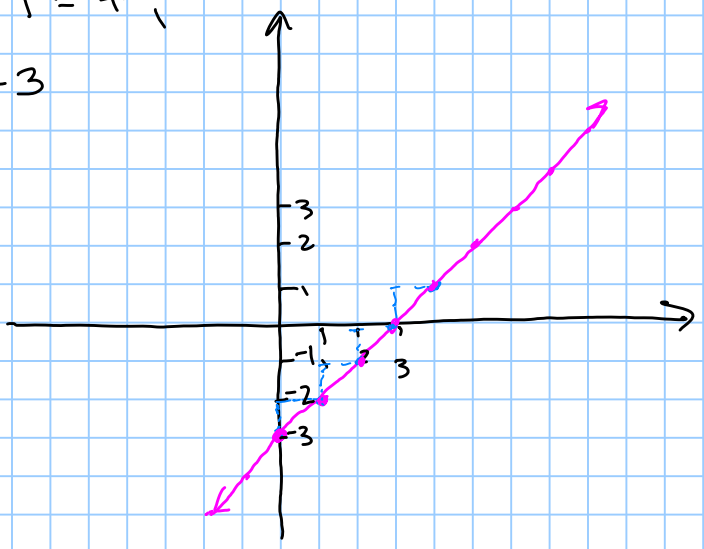
Ex: Graph the equation $y = x - 3$

In $y = mx + b$ form: $y = 1x - 3$

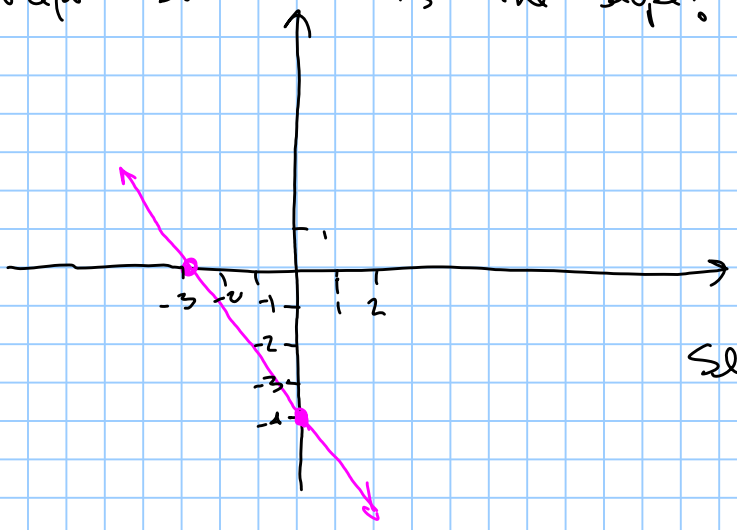
Slope: $m = 1 = +\frac{1}{1}$

y-intercept: $b = -3$

Slope: +



Ex: Graph the line that has y-intercept -4 and x-intercept -3. What is the slope?



Slope: —

vertical change: 4

horizontal change: 3

Slope is $\boxed{-\frac{4}{3}}$

3.5: Equations of Lines (Finding the Equation of a Line)

Ex: Graph the line $2x - 3y = -9$ by writing it in slope-intercept form.

Write as $y = mx + b$: (slope-intercept form)

Solve for y: $2x - 3y = -9$

$-3y = -2x - 9$

$\frac{-3y}{-3} = \frac{-2x}{-3} - \frac{9}{-3}$

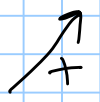
$\boxed{y = \frac{2}{3}x + 3}$

Graph on next page

$y = \frac{2}{3}x + 3$

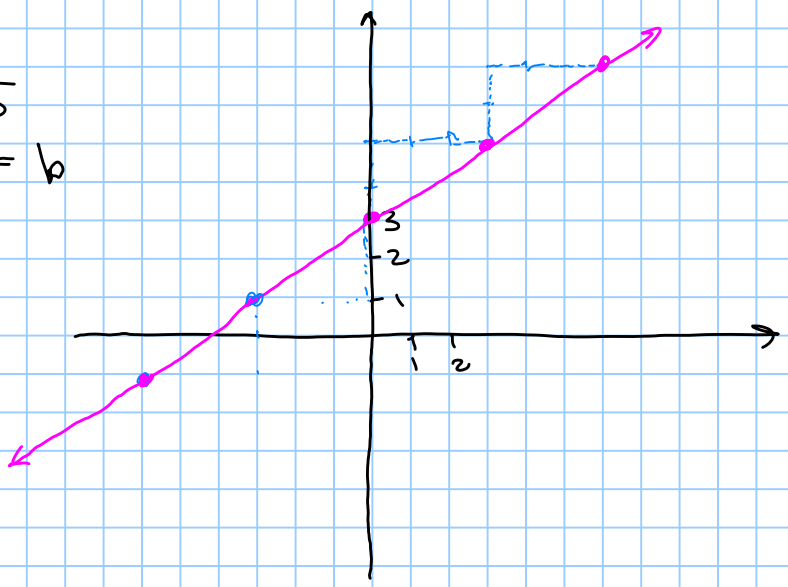
Slope: $m = +\frac{2}{3}$

y-intercept: $3 = b$



vertical: 2

horizontal: 3



Finding the equation of a line given 2 points

Ex: Find the equation of the line containing the points $(2, 5)$ and $(-4, 6)$.

First, find the slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-5}{-4-2} = \frac{1}{-6} = -\frac{1}{6}$

Slope-intercept form: $y = mx + b$

$y = -\frac{1}{6}x + b$ still need b.

Put one of the given points into the equation in place of x and y.

$x = 2,\ y = 5 \Rightarrow \quad 5 = -\frac{1}{6}(2) + b$

Solve for b: $\quad 5 = -\frac{2}{6} + b$

$5 = -\frac{1}{3} + b$

$5 + \frac{1}{3} = b$

$\frac{15}{3} + \frac{1}{3} = b$

$\frac{16}{3} = b$

Put $b = \frac{16}{3}$ into $y = -\frac{1}{6}x + b$:

$$\boxed{y = -\frac{1}{6}x + \frac{16}{3}}$$

Point - Slope Form of a Line

$y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope.

Ex: Find the equation of the line passing through the points $(1, -3)$ and $(-3, -5)$.

Find the slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-3)}{-3 - 1} = \frac{-5+3}{-4} = \frac{-2}{-4} = \frac{1}{2}$

Put this slope, and either of the given points, into the point-slope form:

Using $(1, -3)$:

$m = \frac{1}{2}, x_1 = 1, y_1 = -3 \Rightarrow$

$y - y_1 = m(x - x_1)$

$y - (-3) = \frac{1}{2}(x - 1)$

$y + 3 = \frac{1}{2}(x) + \frac{1}{2}(-1)$

$y + 3 = \frac{1}{2}x - \frac{1}{2}$

$y = \frac{1}{2}x - \frac{1}{2} - 3$

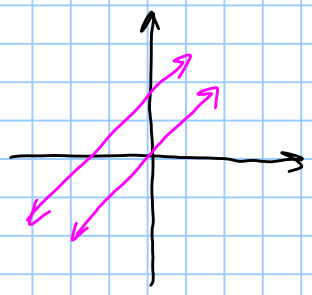
$y = \frac{1}{2}x - \frac{1}{2} - \frac{6}{2}$

$\boxed{y = \frac{1}{2}x - \frac{7}{2}}$

[rearrange until it's in slope-intercept form $y = mx + b$]

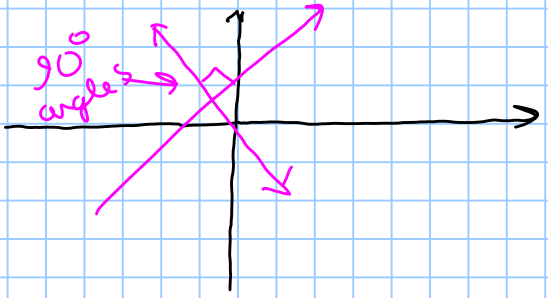
Parallel and Perpendicular Lines

* **Parallel Lines** have the same slope.
(Slopes of parallel lines are equal)



* **Perpendicular Lines**

Perpendicular Lines: intersect at 90° angle



The slopes of perpendicular lines are opposite reciprocals.

Ex: If a line has slope 3, then

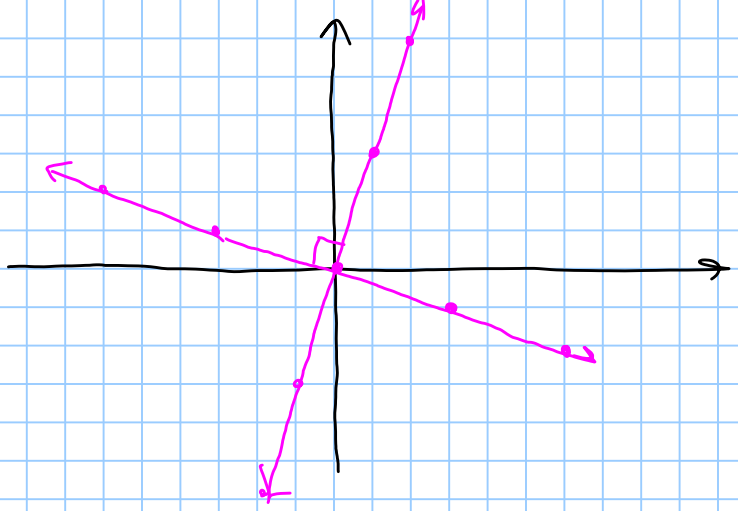
a) all lines parallel to it also have slope 3.

b) all lines perpendicular to it have slope $-\frac{1}{3}$.

See next page

Line 1:
Slope: $m_1 = 3 = +\frac{3}{1}$

Line 2:
slope: $m_2 = -\frac{1}{3}$



Ex: Find the equation of the line that includes the point $(-1,2)$ and is parallel to the line $4x-2y=6$.

Find slope of the given line $4x-2y=6$:

Write in $y=mx+b$ form: $-2y = -4x+6$

$$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{6}{-2}$$

$$y = 2x-3$$

Slope of given line: $m=2$

Slope of desired line: $m=2$ (because they're parallel)

Use point-slope form: $y-y_1 = m(x-x_1)$

$x_1=-1$, $y_1=2$, $m=2$ $\Rightarrow$ $y-2 = 2(x-(-1))$

$y-2 = 2(x+1)$

$y-2 = 2x+2$

$\boxed{y = 2x+4}$

Ex: Find the equation of the line that passes through $(4,5)$ and is perpendicular to $-3x-5y=7$.

Find slope of given line by writing in the form $y=mx+b$:

$$-3x-5y=7$$
$$-5y=3x+7$$
$$\frac{-5y}{-5}=\frac{3x}{-5}+\frac{7}{-5}$$
$$y=-\frac{3}{5}x-\frac{7}{5}$$

Slope of given line: $m_1=-\frac{3}{5}$

Slope of desired line: $m=+\frac{5}{3}$

(because they are perpendicular)

Could use $y-y_1=m(x-x_1)$ as before, or

Use $y=mx+b$: $y=\frac{5}{3}x+b$

$x=4, y=5 \Rightarrow$ $5=\frac{5}{3}(4)+b$

$$5=\frac{20}{3}+b$$
$$5-\frac{20}{3}=b$$
$$\frac{15}{3}-\frac{20}{3}=b$$
$$-\frac{5}{3}=b$$

Equation of desired line: $\boxed{y=\frac{5}{3}x-\frac{5}{3}}$

Prereading Assignment

Read Section 4.1: pp 310-313

Rework example 4: (p 312)

Work similar example: Matched Problem #4 (p 312) (in margin)

Due Monday