4.6: Special Products (cont'd)

Note Title 10/19/2015

Perfect Squares:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Ex: $(x-7)^2 = \boxed{x^2 - 14x + 49}$

Check: $(x-7)(x-7)$

$$= x^2 - 7x - 7x + 49$$

$$= x^2 - 14x + 49$$

4.7: Division of a Polynomial by a Monomial

Ex: $\frac{1}{7} + \frac{3}{7} + \frac{6}{7} + \frac{10}{7}$

$$= \frac{1+3+6+10}{7} = \boxed{\frac{20}{7}}$$

Ex: Divide.

$$\frac{8x^6 - 7x^5 + 4x^3 - 9x + 2}{2x^2}$$

$$= \frac{\cancelto{4}{8}x^6}{\cancelto{1}{2}x^2} - \frac{7x^5}{2x^2} + \frac{\cancelto{2}{4}x^3}{\cancelto{1}{2}x^2} - \frac{9x}{2x^2} + \frac{\cancelto{1}{2}}{\cancelto{1}{2}x^2}$$

$$= \frac{4x^4}{1} - \frac{7x^3}{2} + \frac{2x}{1} - \frac{9}{2x} + \frac{1}{x^2} = \boxed{4x^4 - \frac{7}{2}x^3 + 2x - \frac{9}{2x} + \frac{1}{x^2}}$$

Ex:

$$\frac{6x^5y^3 - 12x^3 + 5xy^2 - 9y^5}{2xy^2}$$

$$= \frac{\overset{3}{\cancel{6}}x^5y^3}{\underset{1}{\cancel{2}}xy^2} - \frac{\overset{6}{\cancel{12}}x^3}{\underset{1}{\cancel{2}}xy^2} + \frac{5xy^2}{2xy^2} - \frac{9y^5}{2xy^2}$$

$$= \frac{3x^4y}{1} - \frac{6x^2}{y^2} + \frac{5}{2} - \frac{9y^3}{2x}$$

$$= \boxed{3x^4y - \frac{6x^2}{y^2} + \frac{5}{2} - \frac{9y^3}{2x}}$$

4.8: Polynomial Long Division

(Dividing a Polynomial by a Polynomial)

Review of Terminology:

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor}\overline{)\ \text{Dividend}} \\ \hline \\ \text{Remainder} \end{array}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$

Example: Divide. $\frac{379}{12}$.

$$\begin{array}{r} 31 \\ 12\overline{)379} \\ -36 \\ \hline 19 \\ -12 \\ \hline 7 \end{array}$$

Check our answer:

$31(12) + 7$

$= 372 + 7$

$= 379$ ✓

$$\begin{array}{r} 31 \\ \times 12 \\ \hline 62 \\ 31 \\ \hline 372 \end{array}$$

$$\frac{379}{12} = \boxed{31 + \frac{7}{12}}$$ usually written $\boxed{31\frac{7}{12}}$

Example: Divide.

$$\frac{3x^2+7x+9}{x+2}$$

Note: We could write $\dfrac{3x^2}{x+2} + \dfrac{7x}{x+2} + \dfrac{9}{x+2}$

but none of these fractions can be reduced.

If your divisor has 2 or more terms, you need <u>polynomial long division.</u>

$\frac{3x^2}{x} \downarrow$ $\quad \frac{x}{x} \downarrow$

$$\begin{array}{r|rrr} & 3x & +\ 1 & \\ \hline x+2 & 3x^2 & +\ 7x & +\ 9 \\ & -(3x^2 & +\ 6x) & \downarrow \\ \hline & 0 & +\ x & +\ 9 \\ & & -(x & +\ 2) \\ \hline & & 0 & +\ 7 \end{array}$$

$\leftarrow 3x(x+2) = 3x^2+6x$

$\leftarrow 1(x+2) = x+2$

$$\frac{3x^2+7x+9}{x+2} = \boxed{3x+1+\frac{7}{x+2}}$$

Note: if $x=10$, this is the same as the previous example

Check:

$(x+2)(3x+1)+7$

$= 3x^2+1x+6x+2+7$

$= 3x^2+7x+9 \quad \checkmark$ ok

Example: Divide $\dfrac{x^3-6x^2+x+14}{x-5}$

$\frac{x^3}{x}$ $\quad$ $\frac{-x^2}{x}$ $\quad$ $\frac{-4x}{x}$

$$
\begin{array}{r|llll}
 & x^2 & -x & -4 & \\
\hline
x-5 & x^3-6x^2 & +x & +14 & \\
 & -(x^3-5x^2) & & & \leftarrow x^2(x-5)=x^3-5x^2 \\
\hline
 & -x^2 & +x & & \\
 & -(-x^2 & +5x) & & \leftarrow -x(x-5)=-x^2+5x \\
\hline
 & & -4x & +14 & \\
 & & -(-4x & +20) & \leftarrow -4(x-5)=-4x+20 \\
\hline
 & & & -6 &
\end{array}
$$

$$\frac{x^3-6x^2+x+14}{x-5}=\boxed{x^2-x-4+\frac{-6}{x-5}}=\boxed{x^2-x-4-\frac{6}{x-5}}$$

Check: $(x-5)(x^2-x-4)-6$

$= x(x^2-x-4)-5(x^2-x-4)-6$

$= x^3-x^2-4x$
$\quad -5x^2+5x+20-6$

$= x^3-6x^2+x+14$ ✓