Homework Qs

Note Title 10/21/2015

4.7 #39 $\dfrac{10a^2 - 15a^2b + 25a^2b^2}{5a^2}$

$$= \frac{10a^2}{5a^2} - \frac{15a^2b}{5a^2} + \frac{25a^2b^2}{5a^2}$$

$$= \frac{2}{1} - \frac{3b}{1} + \frac{5b^2}{1} = \boxed{2 - 3b + 5b^2}$$

(Apparently the back of the book has an error)

4.7 #55 $\dfrac{(x+2)^2 - (x-2)^2}{2x}$

$$= \frac{x^2 + 4x + 4 - (x^2 - 4x + 4)}{2x}$$

$$= \frac{x^2 + 4x + 4 - x^2 + 4x - 4}{2x}$$

$$= \frac{8x}{2x} = \frac{8x}{2x} = \frac{4}{1} = \boxed{4}$$

Scratchwork

$(x+2)^2 = (x+2)(x+2)$

$= x^2 + 2x + 2x + 4$

$= x^2 + 4x + 4$

$(x-2)^2 = (x-2)(x-2)$

$= x^2 - 2x - 2x + 4 = x^2 - 4x + 4$

4.8: Polynomial Long Division (cont'd)

Divide. $\dfrac{8x^3 - 10x^2 + x - 2}{2x - 1}$

$2x(?) = 8x^3$ $\quad 2x(?) = -6x^2$ $\quad 2x(?) = -2x$

$$2x-1 \;\overline{)\; 8x^3 - 10x^2 + x - 2}$$

$$4x^2 - 3x - 1$$

$$\pm\,(8x^3 \mp 4x^2) \quad \leftarrow 4x^2(2x-1)$$

$$-6x^2 + x$$

$$\pm\,(\pm 6x^2 \mp 3x) \quad \leftarrow -3x(2x-1)$$

$$-2x - 2$$

$$\pm\,(\pm 2x \mp 1) \quad \leftarrow -1(2x-1)$$

$$-3$$

$$\frac{8x^3 - 10x^2 + x - 2}{2x - 1} = 4x^2 - 3x - 1 + \frac{-3}{2x-1}$$

or $\quad 4x^2 - 3x - 1 - \dfrac{3}{2x-1}$

Check our answer:

$$(2x-1)(4x^2 - 3x - 1) - 3$$

$$= 8x^3 - 6x^2 - 2x$$
$$\quad - 4x^2 + 3x + 1 - 3$$

$$= 8x^3 - 10x^2 + x - 2 \;\checkmark$$

Important: Arrange terms in order, starting with the largest exponent. Insert "placeholders" (ex: $0x^3$, $0x$) for missing powers of x.

Ex: Divide $\dfrac{3x^4 - 50x^2 - x}{x - 4}$

$$
\begin{array}{r}
3x^3 + 12x^2 - 2x - 9 \\
x-4 \overline{)\,3x^4 + 0x^3 - 50x^2 - x + 0} \\
\ominus\ (-3x^4 \pm 12x^3) \\
\hline
12x^3 - 50x^2 \\
\ominus\ (12x^3 \pm 48x^2) \\
\hline
-2x^2 - x \\
\ominus\ (-2x^2 \pm 8x) \\
\hline
-9x + 0 \\
\ominus\ (-9x \pm 36) \\
\hline
-36
\end{array}
$$

$$\frac{3x^4 - 50x^2 - x}{x - 4} = \boxed{3x^3 + 12x^2 - 2x - 9 - \frac{36}{x-4}}$$

$$(x-4)(3x^3 + 12x^2 - 2x - 9) - 36$$

$$= 3x^4 + 12x^3 - 2x^2 - 9x$$
$$\quad - 12x^3 - 48x^2 + 8x + 36 - 36$$

$$= 3x^4 - 50x^2 - x \quad \checkmark \text{ OK}$$

Ex: Divide. $\dfrac{10x^3 + 21x^2 - 5}{2x^2 + 5x + 2}$

$$2x^2+5x+2 \;\overline{)\; 10x^3 + 21x^2 + 0x - 5}$$

$$5x - 2$$

$$\pm\,(\mp 10x^3 \mp 25x^2 \mp 10x) \quad \leftarrow 5x(2x^2+5x+2)$$

$$-4x^2 - 10x - 5$$

$$\pm\,(\pm 4x^2 \pm 10x \pm 4)$$

$$-1$$

$$\frac{10x^3+21x^2-5}{2x^2+5x+2} = 5x - 2 + \frac{-1}{2x^2+5x+2}$$

$$= 5x - 2 - \frac{1}{2x^2+5x+2}$$

In general, the degree of the remainder will be 1 less than the degree of the divisor

Check it!

4.8 #41) $\dfrac{x^3+64}{x+4}$

$$x+4\overline{)\,x^3+0x^2+0x+64}$$

$$x^2-4x+16$$

$$-(x^3+4x^2)$$

$$-4x^2+0x$$

$$-(-4x^2-16x)$$

$$16x+64$$

$$-(16x+64)$$

$$0$$

$$\frac{x^3+64}{x+4} = \boxed{x^2-4x+16}$$

Check: $(x+4)(x^2-4x+16)$

$= x^3-4x^2+16x$

$+4x^2-16x+64 = x^3+64$ ✓

Chapter 5: Factoring Polynomials

$(x+3)(x+4)$ ← Factored Form

$= x^2 + 7x + 12$ ← Unfactored Form

5.1: The Greatest Common Factor and Factoring by Grouping

To find the Greatest Common Factor (GCF):

* Find the largest number that divides into all the coefficients
* Find the highest power of each variable that is a factor of all the terms. (if a variable shows up in all the terms, choose the smallest exponent)
* multiply the numerical part and the variable part to get the GCF.

Example: Factor out the GCF.

GCF: $3x^2$

$15x^4 - 6x^2$

$= 3x^2\left(\frac{15x^4}{3x^2} - \frac{6x^2}{3x^2}\right)$

$= \boxed{3x^2(5x^2 - 2)}$

Check by distributing: $3x^2(5x^2 - 2) = 15x^4 - 6x^2$ ✓ OK

Ex: Factor.

$48x^3 - 32x$

GCF: $8x$

$= 8x(6x^2 - 4)$

we can still factor 2 out of $6x^2 - 4$. This means we did not use the greatest common factor

$= 8x(2)(3x^2 - 2)$

$= \boxed{16x(3x^2 - 2)}$

Check: $16x(3x^2 - 2) = 48x^3 - 32x$ ✓

OR, if we get the correct GCF at the beginning:

$48x^3 - 32x$

GCF: $16x$

$= \boxed{16x(3x^2 - 2)}$

Ex: Factor.

$24x^3 + 18x^2 - 6$

$= \boxed{6(4x^3 + 3x^2 - 1)}$

Check: $6(4x^3 + 3x^2 - 1)$

$= 24x^3 + 18x^2 - 6$ ✓

Ex: $-30x^4y^3z + 12x^3y^6 - 42x^4y^2z$

GCF: $6x^3y^2$

$= 6x^3y^2\left(\frac{-30x^4y^3z}{6x^3y^2} + \frac{12x^3y^6}{6x^3y^2} - \frac{42x^4y^2z}{6x^3y^2}\right)$

$= \boxed{6x^3y^2(-5xyz + 2y^4 - 7xz)}$

Check it! by multiplying

Factoring by Grouping

Ex: Factor.

$$3x^3 - 7x^2 + 15x - 35$$

$$= (3x^3 - 7x^2) + (15x - 35)$$

$$= x^2(3x - 7) + 5(3x - 7)$$

If the expressions in parentheses are equal, you can factor it out.

$$= \boxed{(3x-7)(x^2+5)}$$

Check: $(3x-7)(x^2+5)$

$= 3x^3 + 15x - 7x^2 - 35$ ✓ ok

Ex: $3xy - 5x - 6y + 10$

$$= (3xy - 5x) + (-6y + 10)$$

$$= x(3y - 5) + 2(-3y + 5)$$

Parentheses don't match! Factor out -2 instead.

$$= x(3y - 5) - 2(3y - 5)$$

$$= \boxed{(3y-5)(x-2)}$$

Also correct: $\boxed{(x-2)(3y-5)}$

Check: $(3y-5)(x-2)$

$= 3xy - 6y - 5x + 10$