

Homework Qs from 5.1

Note Title

10/28/2015

5.1 #17) $21x^2y - 28xy^2$

GCF: $7xy$

$$= \boxed{7xy(3x - 4y)}$$

check: $21x^2y - 28xy^2 \checkmark$

#47) $3xb - 4b - 6x + 8$

GCF: 1

$$= (3xb - 4b) + (-6x + 8) \quad \text{[Factor by grouping]}$$

$$= b(3x - 4) + 2(-3x + 4)$$

← Parentheses don't match
Factor out -2 instead.

$$= b(3x - 4) - 2(3x - 4)$$

$$= \boxed{(3x - 4)(b - 2)}$$

Also correct: $\boxed{(b - 2)(3x - 4)}$

#53) $ax + ay + bx + by + cx + cy$

$$= (ax + ay + bx) + (by + cx + cy)$$

Can only factor out 1.
Dead end.

$$= (ax + ay) + (bx + by) + (cx + cy)$$

$$= a(\underline{x+y}) + b(\underline{x+y}) + c(\underline{x+y})$$

$$= \boxed{(x+y)(a+b+c)} \quad \text{check it!}$$

or

$$ax + ay + bx + by + cx + cy$$

$$= (ax + bx + cx) + (ay + by + cy)$$

$$= x(\underline{a+b+c}) + y(\underline{a+b+c})$$

$$= \boxed{(a+b+c)(x+y)}$$

[Rearrange order]

Ex. Factor $2a^2b - 16a^2 + b - 8$

$$= (2a^2b - 16a^2) + (b - 8)$$

$$= 2a^2(b - 8) + 1(b - 8)$$

$$= \boxed{(b - 8)(2a^2 + 1)}$$

Check: $(b - 8)(2a^2 + 1)$

$$= 2a^2b + b - 16a^2 - 8 \quad \checkmark$$

S.1 # 59 $20x^2 + 4x + 25x + 5$

$$= (20x^2 + 4x) + (25x + 5)$$

$$= 4x(5x + 1) + 5(5x + 1)$$

$$= \boxed{(5x + 1)(4x + 5)}$$

Check: $(5x + 1)(4x + 5)$

$$= 20x^2 + 25x + 4x + 5 \quad \checkmark_{ok}$$

S.1 cont'd:

Ex. $-16x^2y - 4x^2 + 24xy + 6x$

$$= 2x(-8xy - 2x + 12y + 3)$$

$$= 2x[(-8xy - 2x) + (12y + 3)]$$

$$= 2x[-2x(4y + 1) + 3(4y + 1)]$$

$$= 2x[(-2x + 3)(4y + 1)]$$

$$= \boxed{2x(-2x + 3)(4y + 1)}$$

See next page

Another way to factor previous example:

$$\begin{aligned} & -16x^2y - 4x^2 + 24xy + 6x \\ &= (-16x^2y - 4x^2) + (24xy + 6x) \\ &= -4x^2(4y + 1) + 6x(4y + 1) \\ &= (4y + 1)(-4x^2 + 6x) \end{aligned}$$

can factor out 2x

$$= (4y + 1)(2x)(-2x + 3)$$

$$= \boxed{2x(4y + 1)(-2x + 3)}$$

Same as before

Also correct: $-2x(4y + 1)(2x - 3)$

5.2: Factoring Trinomials (when the leading coefficient is 1)

Leading term (of a polynomial): Term with the highest power

Leading coefficient: coefficient of leading term

Ex: $2x^2 - 7x^3 + 12x + 19$

Leading term: $-7x^3$

Degree: 3

Leading coefficient: -7

When factoring polynomials, rearrange it so it is written in order of descending powers (so the leading term is the 1st term)

Example:

Factor.

$$x^2 + 6x + 8$$

$(+)$ signs are the same

$$= (x + 4)(x + 2)$$

Check: $(x + 4)(x + 2)$

$$= x^2 + 2x + 4x + 8$$

$$= x^2 + 6x + 8$$

$$\begin{array}{r} 8 \\ \wedge \\ 1 \cdot 8 \\ 2 \cdot 4 \end{array}$$

Ex.:

$$x^2 + 10x - 24$$

$(-)$ signs are opposite
want a difference of 10
for middle term

$$(x + 12)(x - 2)$$

Check: $x^2 - 2x + 12x - 24$
 $= x^2 + 10x - 24 \checkmark$

$$\begin{array}{r} 24 \\ \wedge \\ 1 \cdot 24 \\ 2 \cdot 12 \\ 3 \cdot 8 \\ 4 \cdot 6 \end{array}$$

Ex.:

$$x^2 + 10x + 24$$

$(+)$ same signs
sum of 10x for middle term

$$(x + 4)(x + 6)$$

Check: $x^2 + 6x + 4x + 24$
 $= x^2 + 10x + 24$

Also correct: $(x + 6)(x + 4)$

$$\begin{array}{r} 24 \\ \wedge \\ 1 \cdot 24 \\ 2 \cdot 12 \\ 3 \cdot 8 \\ 4 \cdot 6 \end{array}$$

Ex.:

$$x^2 - 10x - 24$$

$(-)$ signs are opposite
so want a difference
of 10x for middle term

$$(x - 12)(x + 2)$$

Check: $x^2 + 2x - 12x - 24$
 $= x^2 - 10x - 24 \checkmark$

$$\begin{array}{r} 24 \\ \wedge \\ 1 \cdot 24 \\ 2 \cdot 12 \\ 3 \cdot 8 \\ 4 \cdot 6 \end{array}$$

Ex: $x^2 - 10x + 24$

(+) signs are the same
want a sum of $10x$
for middle term

$$= (x - 6)(x - 4)$$

24
^
1. 24
2. 12
3. 8
4. 6

Check: $x^2 - 4x - 6x + 24$
 $= x^2 - 10x + 24 \checkmark$

Ex: $x^2 + 16x - 36$

(-) signs are opposite
want difference of 16

$$(x + 18)(x - 2)$$

Check: $x^2 - 2x + 18x - 36$
 $= x^2 + 16x - 36 \checkmark$

Also correct: $(x - 2)(x + 18)$

36
^
1. 36
2. 18
3. 12
4. 9
6. 6

Ex: $x^2 + 16x + 36$

(+) same signs
sum of $16x$

It is impossible to find two numbers that multiply to give 36 but have a sum of 16.

This polynomial cannot be factored.

This polynomial is prime.

36
^
1. 36
2. 18
3. 12
4. 9
6. 6

Definition: A polynomial that can only be factored (using integer coefficients) as a product of ± 1 and itself is called prime.

Ex.: $x^2 - 11xy - 12y^2$ (+) opposite signs difference of 11

$$(x - 12y)(x + 1y)$$

Check: $x^2 + 1xy - 12xy - 12y^2$
 $= x^2 - 11xy - 12y^2$ ✓

$$(x - 12y)(x + y)$$

$$\begin{array}{r} 12 \\ \wedge \\ 1 \cdot 12 \\ 2 \cdot 6 \\ 3 \cdot 4 \end{array}$$

5.3: Factoring Trinomials (when the leading coefficient > 1)

Ex.: Factor $2x^2 + 9x + 10$ (+) same signs sum of 9x for middle term

Try $(2x + 1)(x + 10)$

$$2x^2 + 20x + 1x + 10$$

$$= 2x^2 + 21x + 10 \text{ No!}$$

Try $(2x + 10)(x + 1)$

$$= 2x^2 + 2x + 10x + 10$$

$$= 2x^2 + 12x + 10 \text{ No!}$$

Try $(2x + 2)(x + 5)$

$$= 2x^2 + 10x + 2x + 10$$

$$= 2x^2 + 12x + 10 \text{ No!}$$

Try $(2x + 5)(x + 2)$ correct answer

Check: $2x^2 + 4x + 5x + 10$
 $= 2x^2 + 9x + 10$

Ex:

$$4x^2 - 7x - 15$$

$$(4x + 5)(x - 3)$$

Check:

$$4x^2 - 12x + 5x - 15$$

$$= 4x^2 - 7x - 15 \quad \checkmark \text{ OK}$$

$$\begin{array}{cc} 4x^2 & 15 \\ \downarrow & \downarrow \\ x & 3 \end{array}$$

12x

5x

$x \cdot 4x$

$2x \cdot 2x$

$1 \cdot 15$

$3 \cdot 5$