5.2/5.3 Cont'd (Factoring Trinomials)

Note Title 11/2/2015

Factor.

Ex: $-x^2 - 9x - 14$

If the leading coefficient is negative, factor out -1.

$$-x^2 - 9x - 14$$

(+) same signs want sum of $9x$ for middle term

$$= -1(x^2 + 9x + 14)$$

$$= -1(x + 2)(x + 7)$$

$$= \boxed{-(x+2)(x+7)}$$

14
1 · 14
2 · 7

Check: $(x+2)(x+7)$

$= x^2 + 7x + 2x + 14$

$= x^2 + 9x + 14$ ✓

Ex: $2x^3 - 60x^2 + 288x$

(+) same signs want sum of $30x$ for middle term

$$= 2x(x^2 - 30x + 144)$$

$$= \boxed{2x(x - 6)(x - 24)}$$

144
1 · 144
2 · 72
9 · 16
18 · 8
4 · 36
12 · 12
6 · 24
3 · 48

144
2 · 72
2 · 8 · 9
2 · 2 · 4 · 3 · 3
2 · 2 · 2 · 2 · 3 · 3

Check: $(x-6)(x-24)$

$= x^2 - 24x - 6x + 144$

$= x^2 - 30x + 144$ ✓

2
24
6
144 ✓

Ex: $4x^2 + 16x - 9$ (−) opposite signs want a difference for middle term of $16x$

$= (2x - 1)(2x + 9)$

1) GCF factored out? ✓
2) 1st term positive? ✓
3) Arranged in descending order? ✓

Check: $4x^2 + 18x - 2x - 9$
$= 4x^2 + 16x - 9$

$4x^2$: $2x \cdot 2x$, $4x \cdot x$ — 9 : $1 \cdot 9$, $3 \cdot 3$ — $2x$, $18x$

Ex: $18x^2 + 15x - 18$ (−) signs are opposite want a difference of $5x$

$= 3(6x^2 + 5x - 6)$

$= 3(2x + 3)(3x - 2)$

Scratchwork

$6x^2$: $x \cdot 6x$, $2x \cdot 3x$ — 6 : $1 \cdot 6$, $2 \cdot 3$ — $9x$, $4x$

Check: $(2x+3)(3x-2)$
$= 6x^2 - 4x + 9x - 6$
$= 6x^2 + 5x - 6$ ✓
$3(6x^2 + 5x - 6)$
$= 18x^2 + 15x - 18$ ✓

Ex: $-6x^2 + 11xy - 4y^2$ (+) signs the same sum of $11xy$

Factor out -1: $-(6x^2 - 11xy + 4y^2)$

☑ in order?
☑ 1st term positive?
☑ Factor out GCF?

$-(2x - y)(3x - 4y)$

$6x^2$: $6x \cdot x$, $2x \cdot 3x$ — $4y^2$: $4y \cdot y$, $2y \cdot 2y$ — $3xy$, $8xy$

Check: $(2x-y)(3x-4y)$
$6x^2 - 8xy - 3xy + 4y^2$
$6x^2 - 11xy + 4y^2$ ✓

5.4: Difference of 2 squares and perfect Square Trinomials

Recall: $(a+b)(a-b) = a^2 - ab + ab - b^2$

$= a^2 - b^2$

Difference of 2 Squares Factorization:

$$a^2 - b^2 = (a+b)(a-b)$$

Ex: Factor. $x^2 - 36$

$= (x+6)(x-6)$

Check: $(x+6)(x-6)$

$= x^2 - 6x + 6x - 36$

$= x^2 - 36$ ✓

Ex: Factor. $4x^2 - 25$

$= (2x)^2 - (5)^2$ Think of $a = 2x$, $b = 5$

$= (2x+5)(2x-5)$

Check: $(2x+5)(2x-5)$

$= 4x^2 - 10x + 10x - 25$

$= 4x^2 - 25$ ✓

Ex: $100x^2 - 81z^2$ Factor.

$(10x - 9z)(10x + 9z)$

Also correct: $(10x+9z)(10x-9z)$

Ex: Factor. $x^2 + 16$

Try $(x+4)(x-4)$. Check: $x^2 - 4x + 4x - 16$
$= x^2 - 16$ No!

Try $(x+4)(x+4)$

Check: $x^2 + 4x + 4x + 16$
$= x^2 + 8x + 16$ No

$x^2 + 16$ is prime. (cannot be factored)

Important Fact: $x^2 +$ positive is always prime

$a^2 + b^2$ is prime.

Ex: Factor. $32x^2 - 50y^2$

$= 2(16x^2 - 25y^2)$

$= 2(4x + 5y)(4x - 5y)$

Check:

$(4x+5y)(4x-5y)$

$= 16x^2 - 20xy + 20xy - 25y^2$

$= 16x^2 - 25y^2$ ✓

Ex: $x^5 - x$

$= x(x^4 - 1)$

$= x((x^2)^2 - 1^2)$

$= x(x^2 + 1)(x^2 - 1)$

$= x(x^2+1)(x+1)(x-1)$

Ex: Factor.

$$x^2 - 8x + 16$$

$$(x - 4)(x - 4)$$

Check: $x^2 - 4x - 4x + 16$

$= x^2 - 8x + 16$ ✓

$\boxed{(x-4)^2}$ ← final answer

Perfect Square Trinomials:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Ex: $16x^2 - 56xy + 49y^2$

Try $\boxed{(4x - 7y)^2}$

Check: $(4x - 7y)(4x - 7y)$

$16x^2 - 28yx - 28yx + 49y^2$

$= 16x^2 - 56xy + 49y^2$ ✓

Ex: $4x^2 - 20x + 9$

Try: $(2x - 3)^2$

$(2x-3)(2x-3)$

$= 4x^2 - 6x - 6x + 9$

$= 4x^2 - 12x + 9$ No!

correct factorization: $\boxed{(2x-1)(2x-9)}$

Check: $4x^2 - 18x - 2x + 9$

$= 4x^2 - 20x + 9$ ✓