5.5: Sum and Difference of 2 Cubes

Note Title 11/4/2015

Sum and difference of 2 cubes factorization

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Mnemonic Device (for remembering the signs)

M match
O Opposite
P Positive

S Same
O Opposite
A Always
P Positive

Perfect Cubes:

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1000$

Ex: Factor $y^3 - 8$

$= y^3 - 2^3$

$= (y-2)(y^2 + 2y + 2^2)$

$= \boxed{(y-2)(y^2+2y+4)}$

use $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ with $a = y$, $b = 2$

Check: $(y-2)(y^2+2y+4)$

$= y^3 + \cancel{2y^2} + \cancel{4y}$
$\quad - \cancel{2y^2} - \cancel{4y} - 8$

$= y^3 - 8$ ✓ ok

Ex: Factor.

$$27x^3 + 125$$
$$= (3x)^3 + (5)^3$$
$$= (3x + 5)\left((3x)^2 - (3x)(5) + 5^2\right)$$
$$= (3x + 5)(9x^2 - 15x + 25)$$

Common Mistake: $(3x+5)(3x^2 - 15x + 25)$

need to square the 3 as well as the x

Ex: $$128x^4y - 686xy^4$$

GCF: $2xy$

$$= 2xy(64x^3 - 343y^3)$$
$$= 2xy\left[(4x)^3 - (7y)^3\right]$$
$$= 2xy\left[(4x - 7y)\left((4x)^2 + (4x)(7y) + (7y)^2\right)\right]$$
$$= \boxed{2xy(4x - 7y)(16x^2 + 28xy + 49y^2)}$$

$$\begin{array}{r} \overset{6}{49} \\ \times\ 7 \\ \hline 343 \end{array}$$

Ex: $$125w^3 + 8e^3$$

Ex: $2x^4 + 5x^3 - 2x - 5$

$$= (2x^4 + 5x^3) + (-2x - 5)$$
$$= x^3(2x+5) - 1(2x+5)$$
$$= (2x+5)(x^3 - 1)$$
$$= (2x+5)(x^3 - 1^3)$$
$$= (2x+5)\left[(x - 1)(x^2 + 1x + 1^2)\right]$$
$$= \boxed{(2x+5)(x-1)(x^2+x+1)}$$

☆ **Very important:** work all the problems from 5.6: Factoring: A general Review

5.7: Solving Quadratic Equations by Factoring

A quadratic equation (in x) is an equation that can be written in the form $ax^2+bx+c=0$, where a, b, and c are real numbers and $a \neq 0$.

$$ax^2 + bx + c = 0$$ is standard form for a quadratic eqn.

quadratic term (ax^2), linear term (bx), constant term (c)

Zero Product Property of numbers

(sometimes called the Zero Product Theorem)

If the product of two numbers is 0, then at least one of the numbers must be 0.

In other words, if $AB=0$, then $A=0$ or $B=0$.

Steps for solving a quadratic equation:

1) Write eqn in standard form $ax^2+bx+c=0$.
2) Factor the nonzero side.
3) Set each factor equal to 0 (applying the Zero Product Property)
4) Solve the resulting linear equations.
5) Write the solution set.

Recall:
Expressions get simplified (or factored)
Equations get solved.

Ex: Solve.

$$x^2 - 24 = 10x$$

(−) opposite signs difference of 10

$$x^2 - 10x - 24 = 0$$

$$(x-12)(x+2) = 0$$

Check: $(x-12)(x+2)$
$= x^2 + 2x - 12x - 24$
$= x^2 - 10x - 24$ ✓

$x - 12 = 0$ OR $x + 2 = 0$

$\quad +12 \quad +12 \qquad\qquad -2 \quad -2$

$x = 12 \qquad\qquad x = -2$

Solution Set: $\boxed{\{12, -2\}}$

Also correct: $\boxed{\{-2, 12\}}$

Check the solutions to last example:

$$x^2 - 24 = 10x$$

$x=12$:
$$12^2 - 24 = 10(12)$$
$$144 - 24 = 120$$
$$120 = 120 \checkmark$$

$x=-2$:
$$(-2)^2 - 24 = 10(-2)$$
$$4 - 24 = -20$$
$$-20 = -20 \checkmark$$

Ex: Solve.

$$2x^2 - 14x = -24$$

$$2x^2 - 14x + 24 = 0$$

$$2(x^2 - 7x + 12) = 0$$

(+) same signs
sum of 7

$$2(x-4)(x-3) = 0$$

$2 = 0$ never true | OR | $x - 4 = 0$, $x = 4$ | OR | $x - 3 = 0$, $x = 3$

Solution Set: $\boxed{\{3, 4\}}$

Ex: Solve.

$$x^3 = 9x$$

$$x^3 - 9x = 0$$

$$x(x^2 - 9) = 0$$

$$x(x+3)(x-3) = 0$$

$x = 0$ | OR | $x + 3 = 0$, $x = -3$ | OR | $x - 3 = 0$, $x = 3$

Solution Set: $\boxed{\{0, 3, -3\}}$

Also written $\{0, \pm 3\}$

shorthand "plus or minus 3"

Ex:

$$6x^2 - 7x - 5 = 0$$

$$(2x + 1)(3x - 5) = 0$$

(Check it!)

$6x^2$ 5

$x \cdot 6x$ $1 \cdot 5$

$2x \cdot 3x$

$3x$

$10x$

$2x + 1 = 0$

$2x = -1$

$\frac{2x}{2} = \frac{-1}{2}$

$x = -\frac{1}{2}$

$3x - 5 = 0$

$3x = 5$

$\frac{3x}{3} = \frac{5}{3}$

$x = \frac{5}{3}$

$$\boxed{\left\{-\frac{1}{2}, \frac{5}{3}\right\}}$$