

Homework Qs

Note Title

11/9/2015

5.5 #37) $a^6 - b^6$

Try using difference of 2 squares:

$$(a^3)^2 - (b^3)^2$$

$$(a^3 + b^3)(a^3 - b^3)$$

$$(a+b)(a^2-ab+b^2)(a-b)(a^2+ab+b^2)$$

$$(a+b)(a^2-ab+b^2)(a-b)(a^2+ab+b^2)$$

This is a difference of 2 cubes, and also a difference of 2 squares

$$a^6 = (a^2)^3$$

$$a^6 = (a^3)^2$$

What if I started out using difference of 2 cubes?

$$\begin{aligned} a^6 - b^6 &= (a^2)^3 - (b^2)^3 \\ &= (a^2 - b^2)((a^2)^2 + a^2b^2 + (b^2)^2) \\ &= (a+b)(a-b)(a^4 + a^2b^2 + b^4) \end{aligned}$$

This factors into $(a^2-ab+b^2)(a^2+ab+b^2)$, but it's not obvious how

5.6 #61) $y^4 - 1$

$$(y^2)^2 - (1)^2$$

$$(y^2+1)(y^2-1)$$

$$(y^2+1)(y+1)(y-1)$$

5.1 #65) $-4x^8 - 32x^5$

$$= -4x^5(x^3 + 8)$$

$$= -4x^5(x^3 + 2^3)$$

$$= -4x^5(x+2)(x^2-2x+2^2)$$

$$= -4x^5(x+2)(x^2-2x+4)$$

5.6 #69) $(x+2)^4 + 5(x+2)^2$

$$= (x+2)^2 ((x+2)^2 + 5)$$

$$= (x+2)^2 (x^2 + 4x + 4 + 5)$$

$$= \boxed{(x+2)^2 (x^2 + 4x + 9)}$$

This is like (2)

$$y^4 + 5y^2 \text{ with } y = x+2$$

$$y^2(y^2 + 5)$$

Scratchwork:

$$(x+2)^2 = (x+2)(x+2)$$

$$= x^2 + 2x + 2x + 4$$

$$= x^2 + 4x + 4$$

5.6 #71) $(2x+1)^3 - (2x+1)$

$$= (2x+1)((2x+1)^2 - 1)$$

$$= (2x+1)(4x^2 + 4x + 1 - 1)$$

$$= (2x+1)(4x^2 + 4x)$$

$$= (2x+1)(4x)(x+1)$$

$$= \boxed{4x(2x+1)(x+1)}$$

Scratchwork: $(2x+1)^2$

$$(2x+1)(2x+1)$$

$$= 4x^2 + 2x + 2x + 1$$

$$= 4x^2 + 4x + 1$$

5.7 Solving Quadratics (cont'd)

Ex: Solve.

$$-5x^3 + 80x = 0$$

$$-5x(x^2 - 16) = 0$$

$$-5x(x+4)(x-4) = 0$$

$$\begin{array}{l|l|l} -5x = 0 & x+4 = 0 & x-4 = 0 \\ \hline \frac{-5x}{-5} = \frac{0}{-5} & x = -4 & x = 4 \\ x = 0 & & \end{array}$$

$$\begin{array}{r} 16 \\ 5 \overline{) 80} \\ \underline{50} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

Solution

Set:

$$\boxed{\{0, -4, 4\}}$$

or

$$\boxed{\{0, \pm 4\}}$$

Ex: Solve $x^2 = -5x$

$$x^2 + 5x = 0$$

$$x(x+5) = 0$$

$$x = 0 \quad \text{or} \quad \begin{cases} x+5=0 \\ x=-5 \end{cases}$$

Sol'n Set:

$$\{0, -5\}$$

Ex: Solve.

$$4x^2 - 20x + 9 = 0$$

$$(2x-1)(2x-9) = 0$$

$$2x-1=0 \quad | \quad 2x-9=0$$

$$2x=1$$

$$x = \frac{1}{2}$$

$$2x=9$$

$$x = \frac{9}{2}$$

Sol'n Set:

$$\left\{\frac{1}{2}, \frac{9}{2}\right\}$$

scratchwork

Try $(2x-3)^2$
 $(2x-3)(2x-3)$
 $4x^2 - 6x - 6x + 9$
 Nope

$4x^2$		9
$\downarrow$		$\downarrow$
$x \cdot 4x$	not	$1 \cdot 9$
$2x \cdot 2x$		$3 \cdot 3$
		$18x$

Check
 $(2x-1)(2x-9)$
 $= 4x^2 - 18x - 2x + 9$
 $= 4x^2 - 20x + 9$

5.7 #51 $(x+5)^2 = 2x+9$

$$(x+5)(x+5) = 2x+9$$

$$x^2 + 10x + 25 = 2x + 9$$

$$x^2 + 8x + 16 = 0$$

$$(x+4)(x+4) = 0$$

$$x+4=0 \quad \text{or} \quad x+4=0$$

$$x = -4 \quad | \quad x = -4$$

Sol'n Set: $\{-4\}$

5.8: Applications of Quadratic Equations

④

Ex.: The length of a rectangle is 6" more than four times its width. The area of the rectangle is 70 square inches. Find the length and width.

length: $4x + 6$

width: x

length $\xrightarrow{\text{compared to}}$ width $\times$

$$\text{Area of rectangle} = (\text{length})(\text{width})$$

$$70 = (4x + 6)(x)$$

$$70 = x(4x + 6)$$

$$4x^2 + 6x = 70$$

$$4x^2 + 6x - 70 = 0$$

$$2(2x^2 + 3x - 35) = 0$$

$$2(2x - 7)(x + 5) = 0$$

$$2 = 0$$

never true

this factor does not contribute a solution

$$2x - 7 = 0$$

$$2x = 7$$

$$x = \frac{7}{2}$$

$$x + 5 = 0$$

$$x = -5$$

-5 does not make sense for a dimension.

Throw out $x = -5$.

width: $x = \frac{7}{2} = 3\frac{1}{2}$

length: $4x + 6$

$$= 4\left(\frac{7}{2}\right) + 6$$

$$= \frac{28}{2} + 6 = 14 + 6 = 20$$

$$\begin{array}{r} 2x^2 \quad 35 \\ x \cdot 2x \quad \wedge \\ \quad \quad 1 \cdot 35 \\ \quad \quad 5 \cdot 7 \\ \quad \quad 10x \quad 7x \end{array}$$

Check:

$$(2x - 7)(x + 5)$$

$$2x^2 + 10x - 7x - 35$$

$$2x^2 + 3x - 35$$

The width is $3\frac{1}{2}$ inches and the length is 20 inches.

Check on next page:

Check:

$$\text{length: } 20''$$

$$\text{width: } 3\frac{1}{2}'' = \frac{7}{2}''$$

$$4 \text{ times width} = 4\left(\frac{7}{2}\right) = \frac{28}{2} = 14$$

$$6'' \text{ more: } 14 + 6 = 20 \checkmark$$

$$\text{Area} = 20\left(\frac{7}{2}\right) = \frac{20}{1} \left(\frac{7}{2}\right) = \frac{70}{1} = 70 \checkmark$$

Ex.: The height of a triangle is 3 feet less than five times its base. The area of the triangle is 13 ft². Find the base and height.

Base: x

height: $5x - 3$

height $\xrightarrow[\text{to}]{\text{compare}}$ base x

$$\text{Area of triangle} = \frac{1}{2} (\text{base}) (\text{height})$$

$$13 = \frac{1}{2} x (5x - 3)$$

$$2(13) = \cancel{2} \left(\frac{1}{2}\right) x (5x - 3)$$

[multiply both sides by 2
to get rid of fraction]

$$26 = x(5x - 3)$$

$$26 = 5x^2 - 3x$$

$$0 = 5x^2 - 3x - 26$$

$$0 = (5x - 13)(x + 2)$$

$$5x - 13 = 0$$

$$5x = 13$$

$$x = \frac{13}{5}$$

$$x + 2 = 0$$

$$x = -2$$

Throw out. A negative number does not make sense for a dimension

$5x^2$	26
$\wedge$	$\wedge$
$5x \cdot x$	$1 \cdot 26$
$3x$	$2 \cdot 13$
$10x$	

Check

$$\begin{aligned} & (5x - 13)(x + 2) \\ &= 5x^2 + 10x - 13x - 26 \\ &= 5x^2 - 3x - 26 \checkmark \end{aligned}$$

See next page

⑥ Previous example cont'd:

$$\text{base: } x = \frac{13}{5} = 2\frac{3}{5}$$

$$\begin{aligned}\text{height: } 5x - 3 \\ &= 5\left(\frac{13}{5}\right) - 3 \\ &= 13 - 3 = 10\end{aligned}$$

The base is $2\frac{3}{5}$ ft
and the height is 10 ft.

$$\begin{aligned}\text{Check: 5 times base: } 5\left(\frac{13}{5}\right) &= 13 \\ 3 \text{ ft less: } 10 &\checkmark \\ \text{Area} &= \frac{1}{2} \left(\frac{13}{5}\right) \left(\frac{10}{1}\right) = \frac{130}{10} \text{ ft}^2 \\ &= 13 \text{ ft}^2\end{aligned}$$

Ex: The hypotenuse of a right triangle is 2"
longer than the longer leg. The longer leg is 4"
longer than twice the shorter leg. Find the lengths
of all the sides.

Right triangle: longest side = hypotenuse
other two sides: legs

$$\text{hypotenuse: } (2x+4)+2 = 2x+6$$

$$\text{shorter leg: } x$$

$$\text{longer leg: } 2x+4$$

hypotenuse $\xrightarrow{\text{compare to}}$ longer leg
 $\downarrow \text{compare to}$
shorter leg
 x

Pythagorean Theorem:
for right triangles

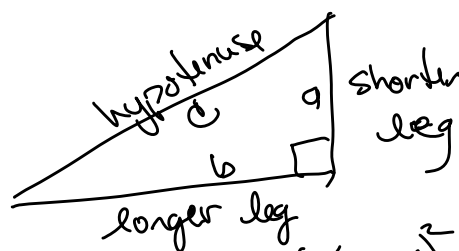
$$a^2 + b^2 = c^2$$

$$x^2 + (2x+4)^2 = (2x+6)^2$$

$$x^2 + 4x^2 + 16x + 16 = 4x^2 + 24x + 36$$

$$5x^2 + 16x + 16 = 4x^2 + 24x + 36$$

See next page



$$\begin{aligned}(2x+4)^2 &= (2x+4)(2x+4) \\ &= 4x^2 + 8x + 8x + 16 \\ &= 4x^2 + 16x + 16 \\ (2x+6)(2x+6) \\ &= 4x^2 + 12x + 12x + 36 \\ &= 4x^2 + 24x + 36\end{aligned}$$

$$5x^2 + 16x + 16 = 4x^2 + 24x + 36$$

$$\text{---}4x^2 \quad \text{---}24x \quad \text{---}36 \quad \text{---}4x^2 \quad \text{---}24x \quad \text{---}36$$

$$x^2 - 8x - 20 = 0$$

$$(x+2)(x-10) = 0$$

$$x = -2 \mid x = 10$$

Throw out

shorter leg: $x = 10$

long leg: $2x + 4$

$$= 2(10) + 4 = 20 + 4 = 24$$

hypotenuse: $2x + 6$

$$= 2(10) + 6 = 20 + 6 = 26$$

The sides are 10", 24, and 26"