5.8: Applications of Quadratic equations (cont'd)

Note Title 11/16/2015

Ex: The height of a triangle is 4 ft less than 3 times its base. The area of the triangle is 42 square feet. Find the base and height.

height $\xrightarrow{\text{compare to}}$ base x

base: x

height: $3x - 4$

$$\text{Area of triangle} = \frac{1}{2}(\text{base})(\text{height})$$

$$42 = \frac{1}{2}(x)(3x-4)$$

multiply by 2: $$84 = x(3x-4)$$

$$84 = 3x^2 - 4x$$

$$0 = 3x^2 - 4x - 84$$

$$0 = (x-6)(3x+14)$$

scratchwork

$3x^2$: $x \cdot 3x$

84: $1\cdot 84$, $2\cdot 42$, $3\cdot 28$, $4\cdot 21$, $14\cdot 6$, $7\cdot 12$

$14x$, $18x$

84: $2\cdot 42$, $2\cdot 6\cdot 7$, $2\cdot 2\cdot 3\cdot 7$

Check: $(x-6)(3x+14)$

$3x^2 + 14x - 18x - 84$

$3x^2 - 4x - 84$ ✓

$14 \times 6 = 84$

$x - 6 = 0$

$x = 6$

or

$3x + 14 = 0$

$3x = -14$

$\frac{3x}{3} = \frac{-14}{3}$

$x = -\frac{14}{3}$

A negative does not make sense for a dimension. Throw out $-\frac{14}{3}$

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Previous example cont'd:

base: $x = 6$ feet

height: $3x - 4 = 3(6) - 4$

$= 18 - 4$

$= 14$ ft

The base is 6 ft and the height is 14 ft.

Check: 3 times base: $3(6) = 18$

4 ft less: $18 - 4 = 14$ ✓

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$
$$= \frac{1}{2}(6\text{ft})(14\text{ft})$$
$$= 3(14)\ \text{ft}\cdot\text{ft}$$
$$= 42\ \text{ft}^2 \checkmark$$

$$\begin{array}{r} 14 \\ \times\ 3 \\ \hline 42 \end{array}$$

Ex: Find two consecutive integers whose product $\underbrace{\text{is}}_{=}$ 27 more than 5 times the larger integer.

1st integer: x

2nd integer: $x+1$

$$\text{Product} = 5\left(\begin{array}{c}\text{larger}\\ \text{integer}\end{array}\right) + 27$$
$$x(x+1) = 5(x+1) + 27$$
$$x^2 + x = 5x + 5 + 27$$
$$x^2 + x = 5x + 32$$
$$x^2 + x - 5x - 32 = 0$$
$$x^2 - 4x - 32 = 0$$

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Previous example cont'd:

$$x^2 - 4x - 32 = 0$$

$$(x-8)(x+4) = 0$$

$$x - 8 = 0 \quad \Big| \quad x + 4 = 0$$

$$x = 8 \quad \Big| \quad x = -4$$

2 possibilities for 1st integer: $x = 8$, $x = -4$

$x=8$) 1st integer: $x = 8$

2nd integer: $x+1 = 8+1 = 9$

The pair of integers is 8, 9.

$x=-4$) 1st integer: $x = -4$

2nd integer: $x+1 = -4+1 = -3$

The pair of integers is $-4, -3$.

$\boxed{\text{The pair of integers is either } 8, 9 \text{ or } -4, -3.}$

Check:

8, 9) Are they consecutive? Yes

Product: $8(9) = 72$

5 times larger: $5(9) = 45$

27 more:

$$\begin{array}{r} 45 \\ +27 \\ \hline 72 \end{array}$$

equal ✓ok

$-4, -3$) Consecutive? Yes ✓

Product: $-4(-3) = 12$

5 times larger: $5(-3) = -15$

27 more: $-15 + 27 = 12$

equal ✓ok

$$\begin{array}{r} 27 \\ -15 \\ \hline 12 \end{array}$$

Ex: Find two consecutive odd integers whose product is one less than six times their sum.

1st integer: x

2nd integer: $x+2$

Note:

Consecutive even integers: $x, x+2, x+4, \ldots$

Consecutive integers: $x, x+1, x+2, \ldots$

Consecutive odd integers: $x, x+2, x+4, \ldots$

$$\text{Product} = 6(\text{sum}) - 1$$

$$x(x+2) = 6(x + x+2) - 1$$

$$x^2+2x = 6x+6x+12-1$$

$$x^2+2x = 12x+11$$

$$x^2+2x-12x-11=0$$

$$x^2-10x-11=0$$

$$(x-11)(x+1)=0$$

$$x-11=0 \quad | \quad x+1=0$$

$$x=11 \quad | \quad x=-1$$

Check: $(x-11)(x+1)$
$= x^2+1x-11x-11$
$= x^2-10x-11$

$x=11$: 1st integer: $x=11$
2nd integer: $x+2=11+2=13$

$x=-1$: 1st integer: $x=-1$
2nd integer: $x+2=-1+2=1$

$\boxed{\text{The integer pair is either } 11, 13 \text{ or } -1, 1.}$

Check it!

Chapter 6: Rational Expressions

6.1: Simplifying Rational Expressions

Recall: <u>Rational Number</u>: can be written as the quotient (ratio, fraction) of two integers.

In other words, a rational number can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

<u>Examples of rational numbers</u>: $\frac{2}{3}$, 2, -3, $\frac{5}{2}$, $\frac{4}{5}$,

$2 = \frac{2}{1}$, $-3 = \frac{-3}{1}$

0, $\frac{13}{5}$, $6\frac{1}{3}$, 7.4, $0.\overline{3}$, 0.29

$0 = \frac{0}{1}$, $6\frac{1}{3} = \frac{19}{3}$, $7.4 = 7\frac{4}{10} = \frac{74}{10}$, $0.\overline{3} = \frac{1}{3}$, $0.29 = \frac{29}{100}$

<u>Examples of irrational numbers</u>:

(irrational = not rational; cannot be written as the ratio of two integers)

$\sqrt{2}$, $\sqrt{3}$, π, e

<u>Rational Expression</u>: can be written as the quotient of two polynomials (with denominator not a constant)

<u>Examples of rational expressions</u>:

$$\frac{x+3}{x-7}, \quad \frac{x^2+9x+2}{3x-7}, \quad \frac{1}{x^2-9},$$

$$\frac{x^4-7x^2+2}{x^3-8x^2+12x-3}$$

To ~~simplify~~ a rational expression, we reduce the fraction by "canceling out" (dividing out) ~~common~~ factors that appear in both the numerator and denominator.

Ex: Reduce $\frac{8}{20}$.

$$\frac{8}{20} = \frac{\cancel{4}\cdot 2}{\cancel{4}\cdot 5} = \boxed{\frac{2}{5}}$$

Ex: Simplify $$\frac{24x^4y^3}{32x^5y} = \boxed{\frac{3y^2}{4x}}$$

Ex: Simplify.

$$\frac{x^2+8x+15}{x^2+5x+6} = \frac{(x+5)\cancel{(x+3)}}{\cancel{(x+3)}(x+2)} = \boxed{\frac{x+5}{x+2}}$$

Note: Restrictions: $x \neq -3,\ x \neq -2$

Ex: Simplify.

$$\frac{x-3}{x^2-7x+12} = \frac{\cancel{x-3}}{\cancel{(x-3)}(x-4)}$$

$$= \boxed{\frac{1}{x-4}}$$

Restrictions: $x \neq 3,\ x \neq 4$

$x-3 \neq 0 \quad x-4 \neq 0$

Ex:

$$\frac{x^2-2x}{4-x^2} = \frac{x(x-2)}{-x^2+4}$$

$$= \frac{x(x-2)}{-1(x^2-4)}$$

$$= \frac{x\cancel{(x-2)}^{1}}{-1(x+2)\cancel{(x-2)}_{1}}$$

$$= \frac{x}{-1(x+2)} = \boxed{-\frac{x}{x+2}}$$