6.4: Solving Equations with Rational Expressions

Note Title 11/23/2015

Example: Solve.

$$\frac{x}{5} + \frac{1}{5} = \frac{4}{5}$$

By inspection: $x = 3$ will make the eqn true:

$$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}. \checkmark$$

Ex: $\frac{x}{8} = \frac{7}{8}$ Solve. $\} x = 7$

Sol'n Set: $\{7\}$.

Note: If 2 fractions are equal, and if they have the same denominator, then their numerators must be equal.

Ex: Solve.

$$\frac{x}{5} - \frac{4x}{3} = \frac{1}{15}$$

LCD: 15

$$\frac{x}{5}\left(\frac{3}{3}\right) - \frac{4x}{3}\left(\frac{5}{5}\right) = \frac{1}{15}$$

$$\frac{3x}{15} - \frac{20x}{15} = \frac{1}{15}$$

$$\frac{-17x}{15} = \frac{1}{15}$$

Multiply both sides by 15: $\left(\frac{\cancel{15}}{1}\right)\frac{-17x}{\cancel{15}} = \frac{1}{\cancel{15}}\left(\frac{\cancel{15}}{1}\right)$

$$-17x = 1$$

$$\frac{-17x}{-17} = \frac{1}{-17}$$

$$x = -\frac{1}{17}$$

$$\boxed{\left\{-\frac{1}{17}\right\}}$$ Sol'n Set

Ex: Solve.

$$\frac{2x-1}{2} - \frac{4x+3}{3} = \frac{x+5}{6}$$

LCD: 6

$$\frac{2x-1}{2}\left(\frac{3}{3}\right) + \frac{-4x-3}{3}\left(\frac{2}{2}\right) = \frac{x+5}{6}$$

$$\frac{6x-3}{6} + \frac{-8x-6}{6} = \frac{x+5}{6}$$

$$\left(\frac{\cancel{6}}{1}\right)\frac{6x-3-8x-6}{\cancel{6}} = \frac{x+5}{\cancel{6}}\left(\frac{\cancel{6}}{1}\right)$$

$$6x-3-8x-6 = x+5$$

$$-2x-9 = x+5$$

$$-2x-x = 5+9$$

$$-3x = 14$$

$$\frac{-3x}{-3} = \frac{14}{-3}$$

$$x = -\frac{14}{3}$$

Solution Set: $\boxed{\left\{-\frac{14}{3}\right\}}$

Ex: $\frac{1}{3x} - \frac{1}{6} = \frac{5}{x}$ Solve.

LCD: $6x$

Restriction: $x \neq 0$

$$\frac{1}{3x}\left(\frac{2}{2}\right) + \frac{-1}{6}\left(\frac{x}{x}\right) = \frac{5}{x}\left(\frac{6}{6}\right)$$

$$\frac{2}{6x} + \frac{-x}{6x} = \frac{30}{6x}$$

$$\left(\frac{\cancel{6x}}{1}\right)\frac{2-x}{\cancel{6x}} = \frac{30}{\cancel{6x}}\left(\frac{\cancel{6x}}{1}\right)$$

$$2-x = 30$$

$$-x = 28$$

$$x = -28$$

Sol'n Set: $\boxed{\{-28\}}$

Ex: Solve.

$$\frac{x}{x+16} = \frac{2}{x-2}$$

LCD: $(x+16)(x-2)$

Restrictions:

$x \neq -16$

$x \neq 2$

$$\frac{x}{x+16}\left(\frac{x-2}{x-2}\right) = \frac{2}{x-2}\left(\frac{x+16}{x+16}\right)$$

$$\frac{x^2-2x}{(x+16)(x-2)} = \frac{2x+32}{(x-2)(x+16)}$$

multiply both sides by $(x+16)(x-2)$, or just set numerators equal.

$$\frac{\cancel{(x-2)}\cancel{(x+16)}}{1}\cdot\frac{x^2-2x}{\cancel{(x+16)}\cancel{(x-2)}} = \frac{2x+32}{\cancel{(x-2)}\cancel{(x+16)}}\cdot\frac{\cancel{(x-2)}\cancel{(x+16)}}{1}$$

$$x^2-2x = 2x+32$$

$$x^2-2x-2x-32=0$$

$$x^2-4x-32=0$$

Factor: $(x+4)(x-8)=0$

$x+4=0$ OR $x-8=0$

$x=-4$ | $x=8$

Solution set: $\boxed{\{-4, 8\}}$

Ex: Solve.

Note: Restrictions $x \neq 3$

$$\frac{2x}{x-3} - \frac{1}{2} = \frac{6}{x-3}$$

LCD: $2(x-3)$

$$\frac{2x}{x-3}\left(\frac{2}{2}\right) + \frac{-1}{2}\left(\frac{x-3}{x-3}\right) = \frac{6}{x-3}\left(\frac{2}{2}\right)$$

$$\frac{4x - x + 3}{2(x-3)} = \frac{12}{2(x-3)}$$

$$\left(\frac{\cancel{2}\cancel{(x-3)}}{1}\right)\frac{3x+3}{\cancel{2}\cancel{(x-3)}} = \frac{12}{\cancel{2}\cancel{(x-3)}}\left(\frac{\cancel{2}\cancel{(x-3)}}{1}\right)$$

$$3x+3 = 12$$

$$3x = 9$$

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

Check it:
(substitute into original eqn)

$$\frac{2x}{x-3} - \frac{1}{2} = \frac{6}{x-3}$$

$x = 3 \Longrightarrow$ $$\frac{2(3)}{3-3} - \frac{1}{2} = \frac{6}{3-3}$$

undefined! $\longrightarrow$ $$\frac{6}{0} - \frac{1}{2} = \frac{6}{0}$$

Cannot divide by 0!

The value 3 does not satisfy the original equation. Throw out $x=3$.

$\boxed{\text{No Solution}}$

Important: When you solve an equation that has variables in the denominator, you must check your solutions to see if they create zero denominators when substituted into the original equation.

(Throw out any "solutions" if they result in zero denominators.

Extraneous Solution: (really not a solution at all)
It's a value generated by a correct solution process, that does not check in the original eqn.

Ex: Solve.

$$\frac{x}{x-2} + 3 = \frac{10}{x-2}$$

LCD: $x-2$
Restrictions: $x \neq 2$

$$\frac{x}{x-2} + \frac{3}{1} = \frac{10}{x-2}$$

$$\frac{x}{x-2} + \frac{3}{1}\left(\frac{x-2}{x-2}\right) = \frac{10}{x-2}$$

$$\left(\frac{\cancel{x-2}}{1}\right)\frac{x+3x-6}{\cancel{x-2}} = \frac{10}{\cancel{x-2}}\left(\frac{\cancel{x-2}}{1}\right)$$

$$4x-6 = 10$$

$$4x = 16$$

$$\frac{4x}{4} = \frac{16}{4}$$

$$x = 4$$

Does this cause zero denominators in the original eqn?
No.

Sol'n Set: $\boxed{\{4\}}$

Chapter 7: Linear Systems

7.1: Solving Linear Systems by Graphing

System of Equations: Group of 2 or more equations.

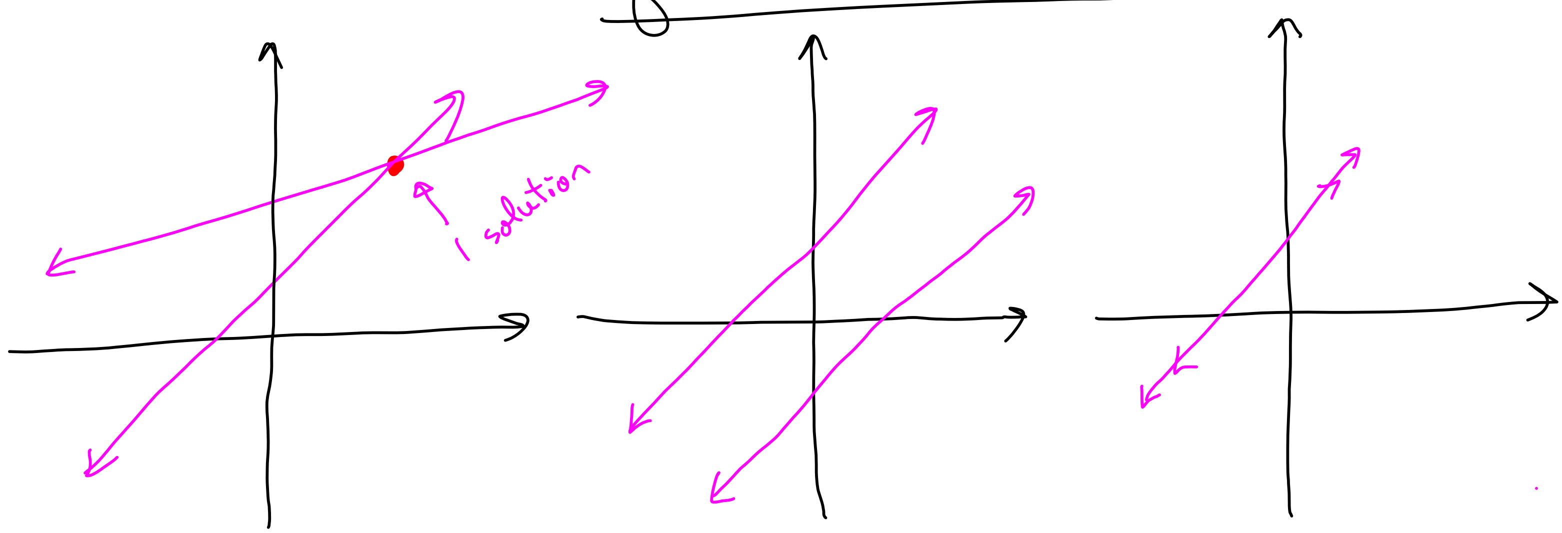
Solution to a system (of eqns): A set of values for the variables that makes all the eqns true.

For us: We will solve systems of 2 linear equations in 2 variables.

Example:
$$\begin{cases} 3x - 4y = 8 \\ -5x + y = 7 \end{cases}$$

Graphs of these are lines

3 possible scenarios for a system of 2 linear equations in 2 variables



Independent System	Inconsistent System	Dependent System
One solution	No Solution	Infinitely many Sol'ns.
lines intersect at a single point.	lines are parallel	Both eqns represent the same line

Question on Test #5

For an inconsistent system, an independent system, and/or a dependent system:

1) State the number of solutions.

2) Describe the graph in words (lines are parallel, lines are the same, lines intersect at one point)

3) Illustrate with an example graph.

Example: Is the ordered pair $(9,-2)$ a solution of the system?

$$2x+5y=8$$
$$3x-2y=23$$

Put $x=9$, $y=-2$ into $2x+5y=8$.

$$2(9)+5(-2)=8$$
$$18-10=8$$
$$8=8 \quad \checkmark \text{ True}$$

Put $x=9$, $y=-2$ into $3x-2y=23$.

$$3(9)-2(-2)=23$$
$$27+4=23$$
$$31=23 \quad \text{False.}$$

$\boxed{\text{No, } (9,-2) \text{ is not a solution of the system.}}$

Is $\left(1, -\frac{2}{3}\right)$ a solution of this system?

$$2x - 3y = 4$$
$$5x + 6y = 1$$

Put $x = 1$, $y = -\frac{2}{3}$ into $2x - 3y = 4$:

$$2(1) - \frac{\cancel{3}}{1}\left(-\frac{2}{\cancel{3}}\right) = 4$$

$$2 + 2 = 4$$
$$4 = 4$$ ✓ok True

Put $x = 1$, $y = -\frac{2}{3}$ into $5x + 6y = 1$:

$$5(1) + \frac{6}{1}\left(-\frac{2}{3}\right) = 1$$

$$5 - \frac{12}{3} = 1$$

$$5 - 4 = 1$$
$$1 = 1$$ ✓ True

Yes, $\left(1, -\frac{2}{3}\right)$ is an solution to the system.

To solve a system by graphing, you graph both equations and look to see where they intersect.

Turn in 7.1 HW for a bonus, or to replace a missed HW.

(But 7.1 HW is not required)

7.2: The Substitution Method

(for solving linear systems)

Steps: (for substitution method)

1. Solve one equation (your choice) for one variable (your choice).
2. Substitute the resulting expression into the <u>other</u> equation.
3. Solve for the remaining variable.
4. Put this value into either equation and solve.
5. Check your answer! It must make <u>both</u> of the original equations true.

Example: Solve the system by substitution.

$$x - y = 9$$
$$2x - 11y = -18$$

1) Solve $x - y = 9$ for x.

$$x = 9 + y$$

2) Substitute $x = 9 + y$ into the other equation:

$$2x - 11y = -18$$

3) Solve for y:

$$2(9 + y) - 11y = -18$$
$$18 + 2y - 11y = -18$$
$$18 - 9y = -18$$
$$-18 \qquad\qquad -18$$
$$-9y = -36$$
$$\frac{-9y}{-9} = \frac{-36}{-9}$$
$$y = 4$$

4) Put $y = 4$ into either equation and solve

Put $y = 4$ into $x - y = 9$.

$$x - 4 = 9$$
$$x = 9 + 4$$
$$x = 13$$

Solution: $\boxed{(13, 4)}$

5) Check it! (see next page)

5) Check: $x - y = 9$

$2x - 11y = -18$

$x - y = 9$

$x = 13,\ y = 4 \Rightarrow 13 - 4 = 9$

$9 = 9$ ✓

$2x - 11y = -18$

$x = 13,\ y = 4 \Rightarrow 2(13) - 11(4) = -18$

$26 - 44 = -18$

$-18 = -18$ ✓

$$\begin{array}{r} 44 \\ -26 \\ \hline 18 \end{array}$$