7.2: The Substitution Method (cont'd.)

Steps for solving by substitution.

1) Solve one equation (your choice) for one variable (your choice).
2) Substitute this expression into the other equation.
3) Solve for the remaining variable.
4) Substitute this value into either equation and solve.
5) Check your solution. (It must make both of the original eqns true)

Example: Solve by substitution.

$$\begin{cases} -4x + 3y = 19 \\ -6x - 5y = 0 \end{cases}$$

1) Solve $-6x - 5y = 0$ for y:

$$-6x = 5y$$

$$\frac{-6x}{5} = \frac{5y}{5}$$

$$-\frac{6}{5}x = y$$

2) Substitute $y = -\frac{6}{5}x$ into $-4x + 3y = 19$:

$$-4x + \frac{3}{1}\left(-\frac{6}{5}x\right) = 19$$

3) Solve

$$-4x - \frac{18}{5}x = 19$$

$$\left(\frac{5}{5}\right)\frac{-4}{1}x - \frac{18}{5}x = 19$$

see next page

$$\left(\frac{5}{5}\right)\frac{-4}{1}x - \frac{18}{5}x = 19$$

$$-\frac{20}{5}x - \frac{18}{5}x = 19$$

$$-\frac{38}{5}x = 19$$

$$\left(-\frac{5}{38}\right)\left(-\frac{38}{5}x\right) = \left(\frac{\cancel{19}^{1}}{1}\right)\left(-\frac{5}{\cancel{38}_{2}}\right)$$

$$x = -\frac{5}{2}$$

4) Put $x = -\frac{5}{2}$ into $-4x + 3y = 19$.

$$-4\left(-\frac{5}{2}\right) + 3y = 19$$

$$+\frac{20}{2} + 3y = 19$$

$$10 + 3y = 19$$

$$3y = 9$$

$$\frac{3y}{3} = \frac{9}{3}$$

$$y = 3$$

Solution: $\boxed{\left(-\frac{5}{2}, 3\right)}$ or $\boxed{\left\{\left(-\frac{5}{2}, 3\right)\right\}}$

step 5

Check:

$-4x + 3y = 19$

$x = -\frac{5}{2},\ y = 3 \Rightarrow \frac{-4}{1}\left(-\frac{5}{2}\right) + 3(3) = 19$

$\frac{20}{2} + 9 = 19$

$10 + 9 = 19$

$19 = 19$ ✓ ok

$-6x - 5y = 0$

$x = -\frac{5}{2},\ y = 3 \Rightarrow -\frac{6}{1}\left(-\frac{5}{2}\right) - 5(3) = 0$

$\frac{30}{2} - 15 = 0$

$15 - 15 = 0$

$0 = 0$ ✓

7.3: The Elimination Method (for solving linear systems)

Steps for the elimination method:

1) multiply one or both equations by an appropriate number, chosen so that a variable is eliminated when the equations are added.

2) Add the equations.

3) Solve for the remaining variable.

4) Find the value of the other variable by
   a) Put the value into either eqn and solve.
   or b) Do elimination again, to find the other variable.

5) Check your solution.

See next page

Example: Solve by elimination (sometimes called the addition method)

$$\begin{cases} 11x - 5y = 2 \\ 3x - y = 1 \end{cases}$$

multiply both sides ~~Multiply~~ of 2nd eqn by -5:

$$\begin{aligned} 11x - 5y &= 2 \longrightarrow & 11x - 5y &= 2 \\ 3x - y &= 1 \xrightarrow{(-5)} & -15x + 5y &= -5 \end{aligned}$$

Add: $-4x + 0y = -3$

$$-4x = -3$$

$$\frac{-4x}{-4} = \frac{-3}{-4}$$

$$x = \frac{3}{4}$$

Now, you could put $x = \frac{3}{4}$ into one of the eqns and solve, as in the substitution method.

OR, do elimination again, on the other variable:

$$\begin{aligned} 11x - 5y &= 2 \xrightarrow{(3)} & 33x - 15y &= 6 \\ 3x - y &= 1 \xrightarrow{(-11)} & -33x + 11y &= -11 \end{aligned}$$

Add: $0x - 4y = -5$

$$-4y = -5$$

$$\frac{-4y}{-4} = \frac{-5}{-4}$$

$$y = \frac{5}{4}$$

Solution: $\boxed{\left(\frac{3}{4}, \frac{5}{4}\right)}$

Check on next page

Check:

$$11x - 5y = 2 \qquad 3x - y = 1$$

$$11\left(\frac{3}{4}\right) - 5\left(\frac{5}{4}\right) = 2 \qquad 3\left(\frac{3}{4}\right) - \frac{5}{4} = 1$$

$$\frac{33}{4} - \frac{25}{4} = 2 \qquad \frac{9}{4} - \frac{5}{4} = 1$$

$$\frac{8}{4} = 2 \qquad \frac{4}{4} = 1$$

$$2 = 2 \checkmark \qquad 1 = 1 \checkmark$$

Ex: Solve.

$$7x - 10y = -13$$
$$-5y - 8x = 3$$

Rewrite:

$$7x - 10y = -13 \longrightarrow 7x - 10y = -13$$
$$-8x - 5y = 3 \xrightarrow{(-2)} 16x + 10y = -6$$

Add: $23x = -19$

$$\frac{23x}{23} = \frac{-19}{23}$$

$$x = -\frac{19}{23}$$

Do elimination again:

$$7x - 10y = -13 \xrightarrow{(8)} 56x - 80y = -104$$
$$-8x - 5y = 3 \xrightarrow{(7)} -56x - 35y = 21$$

Add: $-115y = -83$

$$\frac{-115y}{-115} = \frac{-83}{-115}$$

$$y = \frac{83}{115}$$

Solution: $\boxed{\left(-\frac{19}{23}, \frac{83}{115}\right)}$

Check:

$x = -\frac{19}{23} = -\frac{19}{23}\left(\frac{5}{5}\right) = -\frac{95}{115}$

$$\begin{array}{r} 19 \\ \times 5 \\ \hline 95 \end{array}$$

$$\begin{array}{r} 95 \\ 7 \\ \hline 665 \end{array}$$

$7x - 10y = -13$

$7\left(-\frac{95}{115}\right) - 10\left(\frac{83}{115}\right) = -13$

$-\frac{665}{115} - \frac{830}{115} = -13$

$-\frac{1495}{115} = -13$

$-13 = -13$

$-5y - 8x = 3$

$-5\left(\frac{83}{115}\right) - 8\left(-\frac{95}{115}\right) = 3$

$-\frac{415}{115} + \frac{760}{115} = 3$

$$\begin{array}{r} 760 \\ -415 \\ \hline 345 \end{array}$$

$\frac{345}{115} = 3$

$3 = 3$ ✓

Example: Solve.

$$-x + 6y = 2$$
$$3x - 18y = 5$$

$$\begin{aligned} -x + 6y = 2 &\xrightarrow{(3)} & -3x + 18y &= 6 \\ 3x - 18y = 5 &\longrightarrow & 3x - 18y &= 5 \end{aligned}$$

Add: $0x + 0y = 11$

$0 = 11$ False

$\boxed{\text{No solution}}$

Inconsistent system, lines are parallel

Ex: Solve.

$$2x - 10y = 6$$
$$-5x + 25y = -15$$

$$\begin{aligned} 2x - 10y = 6 &\xrightarrow{(5)} & 10x - 50y &= 30 \\ -5x + 25y = -15 &\xrightarrow{(2)} & -10x + 50y &= -30 \end{aligned}$$

Add: $0x + 0y = 0$

$0 = 0$ True

$\boxed{\text{Dependent system, infinitely many solutions}}$

(lines are the same)

Recognizing Dependent and Inconsistent Systems

* If both variables disappear and leave you with a false statement (ex: $0 = 5$ or $2 = 6$), the system is inconsistent and has no solution.

* If both variables disappear and leave you with a true statement (ex: $0 = 0$, $3 = 3$), the system is dependent and has infinitely many solutions.

7.4: Applications of Linear Systems

Ex: Linda's age is 8 more than twice her daughter's age. The sum of their ages is 101. Find their ages.

Linda's age: x

Daughter's age: y

$$2y + 8 = x$$

2(daughter's age) + 8 = Linda's age

$$x + y = 101$$

2 eqns in 2 variables: $\begin{cases} 2y + 8 = x \\ x + y = 101 \end{cases}$

Put $x = 2y + 8$ into $x + y = 101$:

$$2y + 8 + y = 101$$

$$3y + 8 = 101$$

next page

$$3y + 8 = 101$$
$$3y = 93$$
$$y = \frac{93}{3} = 31$$

Put $y = 31$ into $x + y = 101$:

$$x + 31 = 101$$
$$x = 70$$

$\boxed{\text{Linda is 70 and her daughter is 31.}}$

Ex: Diane has \$0.95 in dimes and nickels. She has a total of 11 coins. How many of each does she have?

number of dimes: d

number of nickels: n

$$d + n = 11 \xrightarrow{(-5)} -5d - 5n = -55$$
$$\$0.10d + \$0.05n = \$0.95 \xrightarrow{(100)} 10d + 5n = 95$$

Add: $5d + 0n = 40$

$$5d = 40$$
$$d = 8$$

Put $d = 8$ into $d + n = 11$:

$$8 + n = 11$$
$$n = 3$$

$\boxed{\text{She has 8 dimes and 3 nickels.}}$