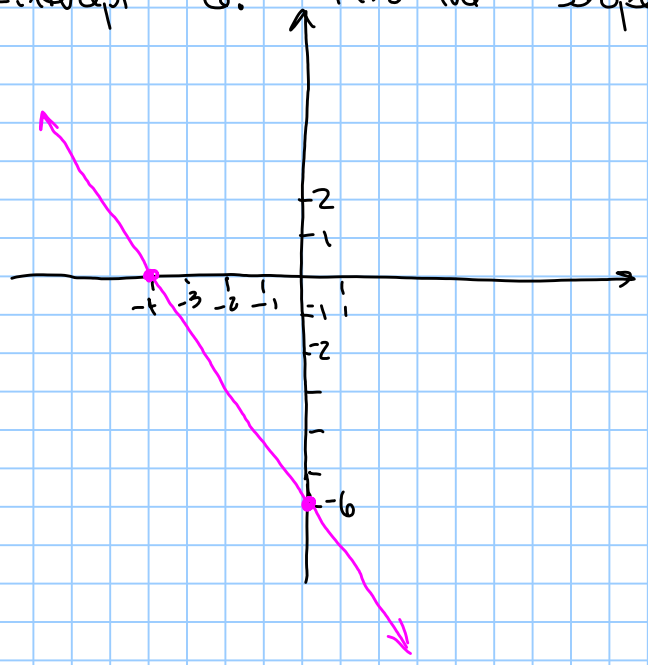
3.4: Graphing Lines Using Slope (cont'd)

Note Title 9/30/2015 ~~9/5/2015~~

Example: Graph the line that has x-intercept -4 and y-intercept -6. Find the slope.



Find slope:

Slope is negative: ↘ (−)

Vertical change: 6

Horizontal change: 4

Slope: $m = -\frac{6}{4} = \boxed{-\frac{3}{2}}$

Or, calculate the slope from the ordered pairs.

x-intercept $-4 \Rightarrow (-4, 0)$

y-intercept is $-6 \Rightarrow (0, -6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-0}{0-(-4)} = \frac{-6}{0+4} = \frac{-6}{4} = \boxed{-\frac{3}{2}}$$

3.5: Finding the equation of a line

Slope-intercept Form of a Line:

$y = mx + b$, where m is the slope and b is the y-intercept.

Example: Graph the line $y = -3x - 4$. (using the slope and y-intercept)

This line is in slope-intercept form: $y = mx + b$

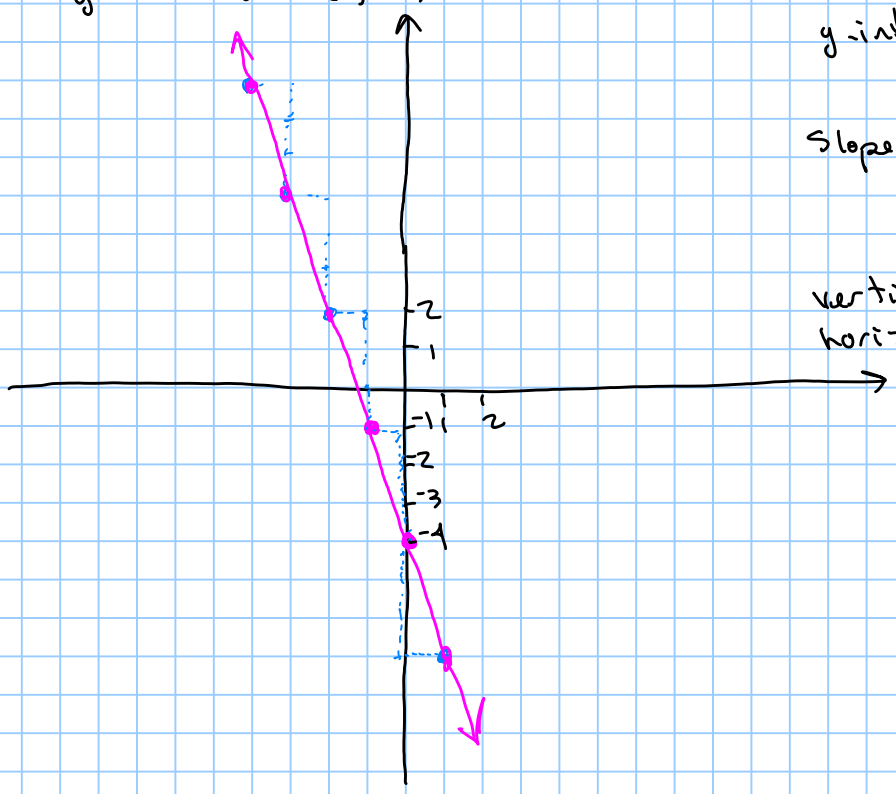
Slope: $m = -3 = -\frac{3}{1}$

y-intercept: $b = -4$

Notice: $x = 0 \Rightarrow y = -3(0) - 4$

$y = 0 - 4$

$y = -4$ Point: $(0, -4)$



y-intercept: -4

Slope: $m = -\frac{3}{1}$

vertical change: 3

horizontal change: 1

Example: Graph the line $2x - 3y = -15$ by writing it in slope-intercept form.

Write as, $y = mx + b$. (solve for y)

$2x - 3y = -15$

$-3y = -2x - 15$

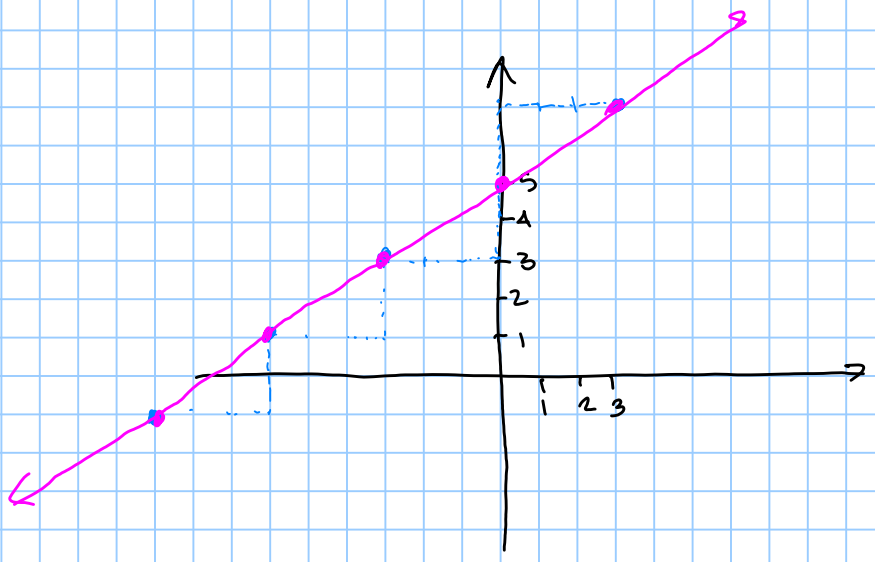
$\frac{-3y}{-3} = \frac{-2x}{-3} - \frac{15}{-3}$

$y = \frac{2}{3}x + 5$

Slope: $m = \frac{2}{3}$

y-intercept: 5 ← b

See next page for graph



Slope: $m = +\frac{2}{3}$

$+$

vertical change: 2
horizontal change: 3

y-intercept: 5

Recall: Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example Find the slope of the line containing the points $(-2, -5)$ and $(4, -8)$.

Let $(x_1, y_1) = (-2, -5)$

$(x_2, y_2) = (4, -8)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8 - (-5)}{4 - (-2)} = \frac{-8 + 5}{4 + 2}$$

$$= \frac{-3}{6}$$

$$= \boxed{-\frac{1}{2}}$$

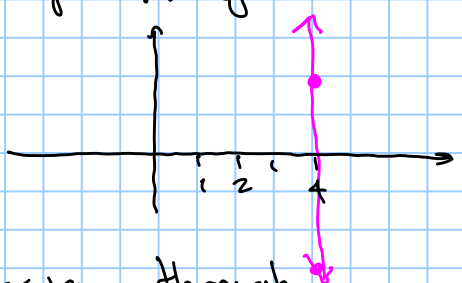
Note: Changing the order of the subtraction gives the same results

$$m = \frac{-5 - (-8)}{-2 - 4} = \frac{-5 + 8}{-6} = \frac{3}{-6} = \boxed{-\frac{1}{2}}$$

Ex: Find the slope of the line passing through $(4, 2)$ and $(4, -3)$.

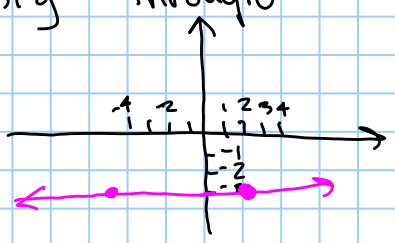
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 2}{4 - 4} = \frac{-5}{0}$$

$\boxed{\text{undefined}}$



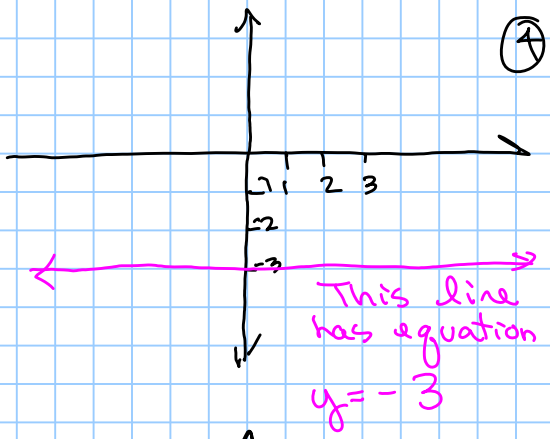
Ex: Find the slope of the line passing through $(2, -3)$ and $(-5, -3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-3)}{-5 - 2} = \frac{-3 + 3}{-7} = \frac{0}{-7} = \boxed{0}$$



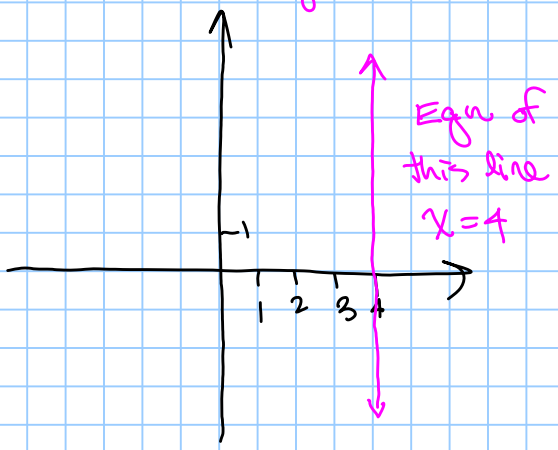
Horizontal lines:

* always have slope = 0
* equation has the form $y = k$ (where k is a constant)



Vertical lines

* always have undefined slope
* equation has the form $x = k$ (k is a constant)



Finding the equation of a line given 2 points:

Ex: Find the equation of the line passing through the points $(-3, 4)$ and $(-1, -2)$.

Find slope 1st: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-4}{-1-(-3)} = \frac{-6}{-1+3} = \frac{-6}{2} = -3$

Final eqn will have the form $y = mx + b$.

$m = -3 \Rightarrow y = -3x + b$ Need to find b.

Put in either of the known points for x and y:

$x = -3, y = 4 \Rightarrow$

$y = -3x + b$

$4 = -3(-3) + b$

$4 = 9 + b$

$4 - 9 = b$

$-5 = b$

Write the equation: $\boxed{y = -3x - 5}$

Point-Slope Form of the Equation of a Line

Point-Slope Form:

$y - y_1 = m(x - x_1)$, where m is the slope, and (x_1, y_1) is a point on the line.

Ex: Find the equation of the line containing the points $(2, 4)$ and $(-3, 2)$.

Find slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-4}{-3-2} = \frac{-2}{-5} = \frac{2}{5}$

Use point-slope form, with $m = \frac{2}{5}$, $(x_1, y_1) = (2, 4)$

$m = \frac{2}{5}$, $x_1 = 2$, $y_1 = 4 \Rightarrow$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{2}{5}(x - 2)$

$y - 4 = \frac{2}{5}x + \frac{2}{5}(-2)$

$y - 4 = \frac{2}{5}x - \frac{4}{5}$

$y = \frac{2}{5}x - \frac{4}{5} + 4$

$y = \frac{2}{5}x - \frac{4}{5} + \frac{20}{5}$

$\boxed{y = \frac{2}{5}x + \frac{16}{5}}$

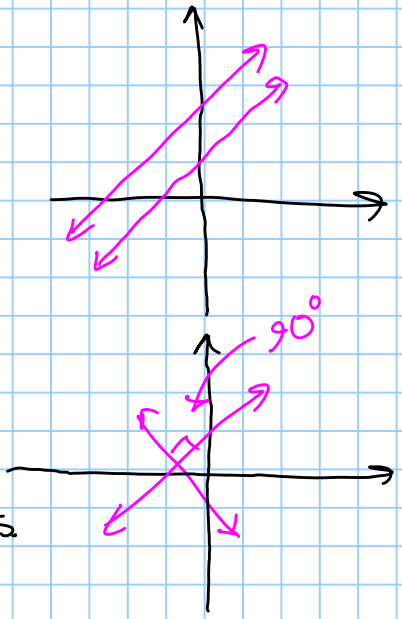
2 forms:

- Point-slope form: $y - y_1 = m(x - x_1)$
- Slope-intercept form: $y = mx + b$

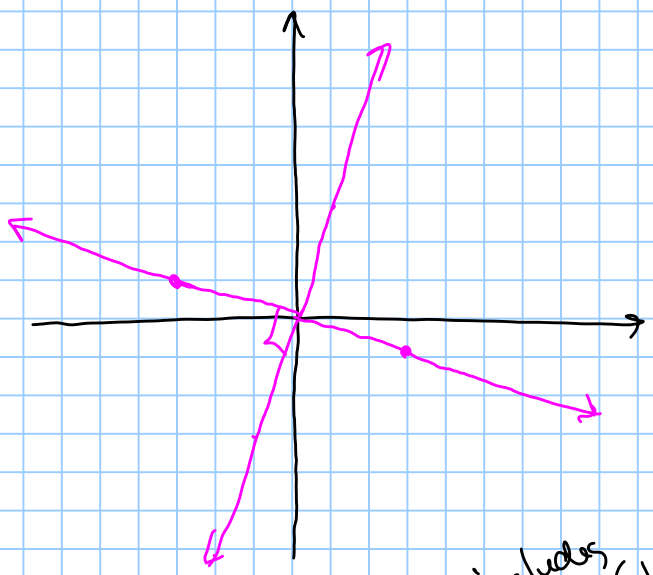
Parallel and perpendicular lines:

* If two lines are parallel, their slopes are equal.

* If two lines are perpendicular, (they intersect at right angles (90°)), their slopes are opposite reciprocals



Ex: Line 1: slope $= 3$
Line 2: slope $= -\frac{1}{3}$



Ex: Find the equation of the line that includes $(1, 2)$
(a) parallel (b) perpendicular to $2x - 3y = 4$.

Find slope of given line: $2x - 3y = 4$

$$-3y = -2x + 4$$
$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{4}{-3}$$
$$y = \frac{2}{3}x - \frac{4}{3}$$

Slope of given line: $m_1 = \frac{2}{3}$

(a) Finding parallel line: Slope of desired line: $m = \frac{2}{3}$
(slopes are the same because they're parallel)

Desired line: $y = mx + b$

$$y = \frac{2}{3}x + b$$

$x = 1, y = 2 \Rightarrow$
$$2 = \frac{2}{3}(1) + b$$

Solve for b: $2 = \frac{2}{3} + b$

$$2 - \frac{2}{3} = b$$
$$\frac{6}{3} - \frac{2}{3} = b$$
$$b = \frac{4}{3}$$

$$\boxed{y = \frac{2}{3}x + \frac{4}{3}}$$ ← answer to part (a)
(this line is parallel to the given line)

(b) For perpendicular line: Slope of given line: $m_1 = \frac{2}{3}$

Slope of desired line:

$$m = -\frac{3}{2}$$

Use $y = mx + b$:

$$y = -\frac{3}{2}x + b$$

$x=1, y=2 \Rightarrow$ $2 = -\frac{3}{2}(1) + b$

$$2 = -\frac{3}{2} + b$$

$$2 + \frac{3}{2} = b$$

$$\frac{4}{2} + \frac{3}{2} = b$$

$$\frac{7}{2} = b$$

Answer to part (b):

$$y = -\frac{3}{2}x + \frac{7}{2}$$

(this line is perpendicular to the given line)

Let's work both problems again, using the point-slope form:

(a) Find the equation of the line passing through $(1,2)$ and parallel to $2x - 3y = 4$.

Find slope of given line by writing it in the form $y = mx + b$:

$$2x - 3y = 4$$

$$-3y = -2x + 4$$

$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{4}{-3}$$

$$y = \frac{2}{3}x - \frac{4}{3}$$ (we already did this)

Slope of given line: $m_1 = \frac{2}{3}$

Slope of desired line: $m = \frac{2}{3}$ (slopes of parallel lines are equal)

Point-slope form: $y - y_1 = m(x - x_1)$

$m = \frac{2}{3}, x_1 = 1, y_1 = 2 \Rightarrow$ $y - 2 = \frac{2}{3}(x - 1)$

$$y - 2 = \frac{2}{3}x + \frac{2}{3}(-1)$$

$$y - 2 = \frac{2}{3}x - \frac{2}{3}$$

$$y = \frac{2}{3}x - \frac{2}{3} + 2$$

$$y = \frac{2}{3}x - \frac{2}{3} + \frac{6}{3}$$

$$\boxed{y = \frac{2}{3}x + \frac{4}{3}}$$ final answer to part (a)

(b) Find the equation of the line passing through $(1,2)$ and perpendicular to $2x-3y=4$.

As before, find slope of given line:
(if you already did it, no need to do it again)

$$-3y = -2x + 4$$

$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{4}{-3}$$

$$y = \frac{2}{3}x - \frac{4}{3}$$

Slope of given line: $m_1 = \frac{2}{3}$

Slope of desired line: $m = -\frac{3}{2}$

(Slopes of perpendicular lines are opposite reciprocals)

Point-slope form: $y - y_1 = m(x - x_1)$

$m = -\frac{3}{2}$, $x=1$, $y=2 \Rightarrow$ $y - 2 = -\frac{3}{2}(x-1)$

$$y - 2 = -\frac{3}{2}(x) - \frac{3}{2}(-1)$$

$$y - 2 = -\frac{3}{2}x + \frac{3}{2}$$

$$y = -\frac{3}{2}x + \frac{3}{2} + 2$$

$$y = -\frac{3}{2}x + \frac{3}{2} + \frac{4}{2}$$

$$\boxed{y = -\frac{3}{2} + \frac{7}{2}}$$

Final answer to part (b)