

Review Problem from 4.1

Ex: Simplify.

$$\begin{aligned}
 & (-2x^3y^2z)^4 (-5x^2y^4z^2)^3 \\
 &= (-2)^4 x^{12} y^8 z^4 (-5)^3 x^6 y^{12} z^6 \\
 &= 16(-125) x^{18} y^{20} z^{10} \\
 &= \boxed{-2000x^{18}y^{20}z^{10}}
 \end{aligned}$$

$$\begin{array}{r}
 125 \\
 \times 16 \\
 \hline
 800 \\
 1000 \\
 \hline
 2000
 \end{array}$$

Recall: Properties of Exponents

$$x^a x^b = x^{a+b}$$

$$(x^a)^b = x^{ab}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

4.2: Division with Exponents and Negative Exponents

Definition of a negative exponent:

$$x^{-n} = \frac{1}{x^n}$$

Note: $x^{-n} = \frac{x^{-n}}{1} = \frac{1}{x^n}$

Also: $\frac{1}{x^{-n}} = \frac{x^n}{1}$

Note: $x^{-1} = \frac{x^{-1}}{1} = \frac{1}{x^1} = \frac{1}{x}$

why?

$$\begin{aligned}
 \frac{1}{x^{-n}} &= \frac{1}{\frac{1}{x^n}} = 1 \cdot \frac{x^n}{1} \\
 &= \frac{1}{1} \cdot \frac{x^n}{1} = \frac{x^n}{1} = x^n
 \end{aligned}$$

Examples: In all these examples, Simplify.
Write answers so that variables have positive exponents.

$$\underline{\text{Ex:}} \quad 7^{-2} = \frac{7^{-2}}{1} = \frac{1}{7^2} = \boxed{\frac{1}{49}}$$

$$\underline{\text{Ex:}} \quad -3^{-4} = -\frac{3^{-4}}{1} = -\frac{1}{3^4} = \boxed{-\frac{1}{81}}$$

$$\underline{\text{Ex:}} \quad (-2)^{-4} = \frac{(-2)^{-4}}{1} = \frac{1}{(-2)^4} = \boxed{\frac{1}{16}}$$

$$\underline{\text{Ex:}} \quad \left(\frac{3}{5}\right)^{-3} = \frac{3^{-3}}{5^{-3}} = \frac{5^3}{3^3} = \boxed{\frac{125}{27}}$$

$$\underline{\text{or}} \quad \left(\frac{3}{5}\right)^{-3} = \left(\frac{5}{3}\right)^3 = \frac{5^3}{3^3} = \boxed{\frac{125}{27}}$$

$$\underline{\text{Ex:}} \quad \frac{1}{10^{-3}} = \frac{10^3}{1} = \frac{1000}{1} = \boxed{1000}$$

$$\underline{\text{Ex:}} \quad 2x^{-5} = \frac{2x^{-5}}{1} = \boxed{\frac{2}{x^5}}$$

$$\underline{\text{Ex:}} \quad (2x^2y)^{-5} = \frac{(2x^2y)^{-5}}{1} = \frac{2^{-5}x^{-10}y^{-5}}{1} = \frac{1}{2^5x^{10}y^5}$$
$$= \boxed{\frac{1}{32x^{10}y^5}}$$

Ex:

$$\frac{x^{-4} y^{-3} z^2}{x^5 y^{-6} z^{-3}} = \frac{z^2}{x^4 y^3} \frac{y^6 z^3}{x^5}$$

$$= \frac{z^2 y^6 z^3}{x^4 y^3 x^5} = \frac{y^6 z^5}{x^9 y^3} = \boxed{\frac{y^3 z^5}{x^9}}$$

Note:

$$\frac{y^3}{y^{12}} = \boxed{\frac{1}{y^9}}$$

Ex:

$$\left(\frac{-2x^{-4}y}{z^5} \right)^{-3} = \frac{(-2)^{-3} x^{12} y^{-3}}{z^{-15}} = \frac{x^{12}}{(-2)^3 y^3} z^{15}$$

$$= \frac{x^{12} z^{15}}{-8 y^3} = \boxed{-\frac{x^{12} z^{15}}{8 y^3}}$$

Ex:

$$\frac{-3x^{-3} (x^{-1} y^3 z^{-2})^{-5}}{(3x^2 y^4)^{-2} (2x y^{-3} z)^4} = \frac{-3x^{-3} x^5 y^{-15} z^{10}}{(-3)^{-2} x^{-4} y^{-8} \cdot 2^4 x^4 y^{-12} z^4}$$

$$= \frac{-3 x^5 z^{10} (-3)^2 x^4 y^8 y^{12}}{x^3 y^{15} \cdot 2^4 x^4 z^4}$$

$$= \frac{-3(9) x^9 y^{20} z^{10}}{16 x^7 y^{15} z^4} = \boxed{-\frac{27 x^2 y^5 z^6}{16}}$$

Recall:

$$\boxed{x^0 = 1 \text{ as long as } x \neq 0}$$

Ex:

$$\left(-\frac{7251}{\pi} \right)^0 = 1$$

$$\text{Ex: } (5x^2 y z^{13})^0 = 1$$

as long as $x \neq 0, y \neq 0, z \neq 0$

Scientific Notation: In scientific notation, we write numbers so they have exactly 1 digit to the left of the decimal point, and are multiplied by a power of 10.

Big numbers (greater than) : positive powers of 10

Small numbers (less than) : negative powers of 10

Ex: write 12,345 in scientific notation.

$$1.2345 \times 10^4$$

1.2345
4 slots

Ex: write 0.000073691 in scientific notation.

$$7.3691 \times 10^{-5}$$

0.000073691
5 slots

Ex: write 2.35×10^7 without

2350000000000000000

$$23,500,000$$

scientific notation.
most calculators
would write this
as
 $2.35E7$
Prev example:
 $7.3691E-5$

4.3: Operations with Monomials

Polynomial: expression composed of terms added together,
in which each term has variables to
positive integer powers only

Examples of polynomials: $x^4 + 3x^2 - 7$

$$x^4 + y^3 - 8y$$

Not polynomials: $x^{-4} + x^{-3} + 2x^{-1}$

$$\frac{1}{x^4} - \frac{3}{x^2}$$

$$\sqrt{x^4 + x^7}$$

Same as
 $x^{-4} - 3x^{-2}$

Degree of a polynomial in 1 variable:

Degree = highest power

Ex: What degree is $2x^4 - 3x^5 + x^2$?

Degree 5:

Special terms for certain polynomials:

Polynomial with 3 terms: trinomial

Polynomial with 2 terms: binomial

Polynomial with 1 term: monomial