Homework Qs

Note Title 10/21/2015

4.6 #59

$$(2x+3)^2 - (4x-1)^2$$

$$= 4x^2+12x+9-(16x^2-8x+1)$$

$$= 4x^2+12x+9-16x^2+8x-1$$

$$= \boxed{-12x^2+20x+8}$$

Scratchwork

$$(2x+3)^2 = (2x+3)(2x+3)$$

$$= 4x^2+6x+6x+9$$

$$= 4x^2+12x+9$$

$$(4x-1)^2 = (4x-1)(4x-1)$$

$$= 16x^2-4x-4x+1$$

$$= 16x^2-8x+1$$

4.8 Polynomial Long Division (Cont'd)

Example: Divide $\dfrac{8x^3 - 10x^2 + x - 2}{2x - 1}$

$2x(?) = 8x^3$ $\quad 2x(?) = -6x^2$ $\quad 2x(?) = -2x$

$$
\begin{array}{r|llll}
 & 4x^2 & -3x & -1 & \\
\hline
2x - 1 & 8x^3 & -10x^2 & +x & -2 \\
- & (8x^3 & -4x^2) & & \leftarrow 4x^2(2x-1) \\
\hline
 & & -6x^2 & +x & \\
- & & (-6x^2 & +3x) & \leftarrow -3x(2x-1) \\
\hline
 & & & -2x & -2 \\
- & & & (-2x & +1) \quad \leftarrow -1(2x-1) \\
\hline
 & & & & -3
\end{array}
$$

$$\frac{8x^3 - 10x^2 + x - 2}{2x - 1} = \boxed{4x^2 - 3x - 1 + \frac{-3}{2x-1}}$$

$$= \boxed{4x^2 - 3x - 1 - \frac{3}{2x-1}}$$

Check answer: $(2x-1)(4x^2 - 3x - 1) - 3$

$= 8x^3 - 6x^2 - 2x$
$\quad - 4x^2 + 3x + 1 - 3$

$= 8x^3 - 10x^2 + x - 2$ ✓ ok

Important: Arrange terms in order, starting with the largest exponent.

✱ Insert "placeholders" (example: $0x^3$, $0x$) for missing powers of x.

Ex: Divide. $\dfrac{3x^4-50x^2-x}{x-4}$

$x(?)=3x^4$ $\quad$ $x(?)=12x^3$ $\quad$ $x(?)=-2x^2$ $\quad$ $x(?)=-9x$

$$
\begin{array}{r|l}
 & 3x^3 + 12x^2 - 2x - 9 \\
\hline
x-4 & 3x^4 + 0x^3 - 50x^2 - x + 0 \\
 & -(3x^4 - 12x^3) \\
\hline
 & 12x^3 - 50x^2 \\
 & -(12x^3 - 48x^2) \\
\hline
 & -2x^2 - x \\
 & -(-2x^2 + 8x) \\
\hline
 & -9x + 0 \\
 & -(-9x + 36) \\
\hline
 & -36
\end{array}
$$

$$\frac{3x^4-50x^2-x}{x-4} = \boxed{3x^3+12x^2-2x-9+\frac{-36}{x-4}}$$

$$= \boxed{3x^3+12x^2-2x-9-\frac{36}{x-4}}$$

Check: $(x-4)(3x^3+12x^2-2x-9)-36$

$= 3x^4 + \cancel{12x^3} - 2x^2 - 9x$

$\quad - \cancel{12x^3} - 48x^2 + 8x \cancel{+36} \cancel{-36}$

$= 3x^4 - 50x^2 - x$ ✓ok

Example:

Divide $\dfrac{10x^3 + 21x^2 - 5}{2x^2 + 5x + 2}$

$2x^2(?) = 10x^3$ $\qquad$ $2x^2(?) = -4x^2$

$$
\begin{array}{r}
5x - 2 \\
2x^2 + 5x + 2 \enclose{longdiv}{10x^3 + 21x^2 + 0x - 5} \\
-(10x^3 + 25x^2 + 10x) \\
\hline
-4x^2 - 10x - 5 \\
-(-4x^2 - 10x - 4) \\
\hline
-1
\end{array}
$$

In general, the degree of the remainder will be 1 less than the degree of the divisor.

(so if the divisor is 1st degree, we expect the remainder to be a constant

(if the divisor is 2nd degree, we expect the remainder to be 1st degree)

$$\frac{10x^3 + 21x^2 - 5}{2x^2 + 5x + 2} = \boxed{5x - 2 + \frac{-1}{2x^2 + 5x + 2}} = \boxed{5x - 2 - \frac{1}{2x^2 + 5x + 2}}$$

Check it!

Chapter 5: Factoring Polynomials

$(x+3)(x+4)$ ⟵ Factored form

$= x^2 + 7x + 12$ ⟵ unfactored form

5.1: The Greatest Common Factor and Factoring by Grouping

To find the Greatest Common Factor (GCF)

* Find the largest number that divides into all the coefficients.
* Find the highest power of each variable that is a factor of all the terms.
  (if a variable shows up in all the terms, choose the smallest exponent)
* To get the GCF, multiply the numerical part and the variable part together.

Ex: Factor out the GCF.

$15x^4 - 6x^2$

GCF: $3x^2$

$= 3x^2\left(\frac{15x^4}{3x^2} - \frac{6x^2}{3x^2}\right)$

$= \boxed{3x^2(5x^2 - 2)}$

Check: $3x^2(5x^2-2)$

$= 15x^4 - 6x^2$ ✓

Factor.

Ex: $48x^3 - 32x$ ~~G~~CF: $8x$

$= 8x(6x^2 - 4)$

We can still factor 2 out of $6x^2 - 4$, so we didn't get the greatest common factor.

$8x(2)(3x^2 - 2)$

$\boxed{16x(3x^2 - 2)}$

We could have factored out GCF $= 16x$ the first time.

$48x^3 - 32x$

$= \boxed{16x(3x^2 - 2)}$

Ex: Factor.

$-30x^4y^3z^2 + 12x^3y^6 - 42x^4y^2z$ GCF: $6x^3y^2$

$= 6x^3y^2\left(\frac{-30x^4y^3z^2}{6x^3y^2} + \frac{12x^3y^6}{6x^3y^2} - \frac{42x^4y^2z}{6x^3y^2}\right)$

$= \boxed{6x^3y^2(-5xyz^2 + 2y^4 - 7xz)}$ Check it!

Factoring by Grouping:

GCF: 1

Example: Factor.

$$3x^3 - 7x^2 + 15x - 35$$
$$= (3x^3 - 7x^2) + (15x - 35)$$
$$= x^2(3x - 7) + 5(3x - 7)$$
$$= \boxed{(3x-7)(x^2+5)}$$

(we want the parentheses to match)

Check: $3x^3 + 15x - 7x^2 - 35$ ✓

could write: $\boxed{(x^2+5)(3x-7)}$

Ex: $3x^2 + 2xy - 3xy - 2y^2$

$$= (3x^2 + 2xy) + (-3xy - 2y^2)$$
$$= x(3x + 2y) - y(3x + 2y)$$
$$= \boxed{(3x+2y)(x-y)}$$

Check: $(3x+2y)(x-y)$

$= 3x^2 - 3xy + 2xy - 2y^2$ ← same as original!

$= 3x^2 - xy - 2y^2$