

5.1: Greatest Common Factor and Factoring by Grouping (cont'd)

Note Title

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$$\text{Ex: } 4x^3 - 3x^2 - 36x + 27$$
$$= (4x^3 - 3x^2) + (-36x + 27)$$

$$= x^2(4x - 3) + 9(-4x + 3)$$

$$= x^2(\underline{4x - 3}) - \underline{9(4x - 3)}$$

$$= (4x - 3)(x^2 - 9)$$
$$= \boxed{(4x - 3)(x + 3)(x - 3)}$$

[parentheses don't match]

[Pull out -9 instead]

$$\text{Check: } (4x - 3)(x^2 - 9)$$
$$= 4x^3 - 36x - 3x^2 + 27 \checkmark$$

Ex: Factor.

$$2a^2b - 16a^2 + b - 8$$

$$= (2a^2b - 16a^2) + (b - 8)$$

$$= 2a^2(\underline{b - 8}) + 1(\underline{b - 8})$$

$$= \boxed{(b - 8)(2a^2 + 1)}$$

$$\text{Check: } 2a^2b + b - 16a^2 - 8 \checkmark$$

Also correct: $\boxed{(2a^2 + 1)(b - 8)}$

Ex. $7xy^2 - 20x + 2y^2 - 70x^2$

$$= (7xy^2 - 20x) + (2y^2 - 70x^2)$$

$$= x(7y^2 - 20) + 2(y^2 - 35x^2)$$

[Parentheses don't match;
switch order of 2nd and
3rd terms]

$$7xy^2 + 2y^2 - 20x - 70x^2$$

$$= (7xy^2 + 2y^2) + (-20x - 70x^2)$$

$$= y^2(7x+2) - 10x(2+7x)$$

$$= y^2(7x+2) - 10x(7x+2)$$

$$= \boxed{(7x+2)(y^2 - 10x)}$$

Check it!

5.2: Factoring Trinomials (when the leading
coefficient is 1)

Leading term (of a polynomial): Term with the
highest power.

Leading coefficient: Coefficient of the leading term.

Ex. $2x^2 - 8x^3 + 11x - 18$

Leading term: $-8x^3$

Degree: 3 $\leftarrow$ highest power

Leading coefficient: -8

To factor a polynomial, we want to write it in order of descending powers (so leading term will be listed first)

Ex: Factor.

$$x^2 + 6x + 8$$

$$(x + 2)(x + 4)$$

(+) signs are the same
want a sum of 6x for middle

First
Outer
Inner
Last

Check: $(x+2)(x+4)$
 $= x^2 + 4x + 2x + 8$
 $= x^2 + 6x + 8 \checkmark$

Also correct: $(x+4)(x+2)$

① Ex: $x^2 + 10x - 24$

$$(x - 2)(x + 12)$$

Check: $x^2 + 12x - 2x - 24$
 $= x^2 + 10x - 24 \checkmark$

(-) signs are opposite
want a difference of 10x for middle term

24

^

1 · 24

2 · 12

3 · 8

4 · 6

② Ex: $x^2 - 10x + 24$

$$(x - 6)(x - 4)$$

Check: $x^2 - 4x - 6x + 24$
 $= x^2 - 10x + 24 \checkmark$

(+) signs are the same
want a sum of 10x for middle term

24

^

1 · 24

2 · 12

3 · 8

4 · 6

②

Ex:

$$x^2 - 10x - 24$$

$$= (x + 2)(x - 12)$$

check: $x^2 - 12x + 2x - 24$
 $= x^2 - 10x - 24$

(-) signs are opposite
 want a difference of 10x for middle term

24
 ^
 1.24
2.12
 3.8
 4.6

③

Ex:

$$x^2 + 10x + 24$$

$$(x + 4)(x + 6)$$

check: $x^2 + 6x + 4x + 24$
 $= x^2 + 10x + 24$

(+) same signs
 want sum of 10

Ex:

$$x^2 + 16x - 36$$

$$(x - 2)(x + 18)$$

check: $x^2 + 18x - 2x - 36$
 $= x^2 + 16x - 36$

(-) signs are opposite
 want a difference of 16x for middle term

36
 ^
 1.36
2.18
 3.12
 4.9
 6.6

Ex:

$$x^2 - 11xy - 12y^2$$

$$(x - 12y)(x + y)$$

check: $x^2 + 1xy - 12xy - 12y^2$
 $= x^2 - 11xy - 12y^2$ ✓

$$(x - 12y)(x + y)$$

(-) want a difference of 11
 signs are opposite

12
 ^
1.12
 2.6
 3.4

Ex: $x^2 - 11xy + 12y^2$ (+) same signs want a sum of 11

12
1
1.12
2.6
3.4

It's impossible to find 2 numbers that give us a product of 12 but a sum of 11

This polynomial cannot be factored.

This polynomial is prime.

Prime polynomial: cannot be factored except by factoring out 1 or -1.

5.3: Factoring Trinomials: (leading coefficient > 1)

Ex: $2x^2 + 9x + 10$ (+) same signs want sum of 9x for middle term

Try $(2x + 1)(x + 10)$

check: $2x^2 + 20x + 1x + 10$
 $= 2x^2 + 21x + 10$ No!

$2x^2$ 10
1
1.10
 $2x \cdot x$ 2.5

Try $(2x + 10)(x + 1)$

check: $= 2x^2 + 2x + 10x + 10$
 $= 2x^2 + 12x + 10$ No!

Try $(2x + 2)(x + 5)$

check: $2x^2 + 10x + 2x + 10$
 $= 2x^2 + 12x + 10$ No!

Try $(2x + 5)(x + 2)$

check: $2x^2 + 4x + 5x + 10 = 2x^2 + 9x + 10$ ✓ ok