5.2/5.3 Cont'd (Factoring Trinomials)

Note Title 11/2/2015

Ex: $2x^2 + 18x + 28$

(+) same signs, want sum of 9x for middle terms

☑ terms in descending order?

☑ leading coefficient positive?

☑ GCF factored out?

$= 2(x^2 + 9x + 14)$

$= \boxed{2(x+7)(x+2)}$

14
1 · 14
2 · 7

Check: $(x+7)(x+2)$

$= x^2 + 2x + 7x + 14$

$= x^2 + 9x + 14$ ✓

Ex: $-x^2 + 14x - 40$

(+) same signs, want sum of 14x

[Factor out −1 to make the leading coefficient positive]

$= -1(x^2 - 14x + 40)$

$= -1(x-4)(x-10)$

40
1 · 40
2 · 20
4 · 10
5 · 8

Check: $(x-4)(x-10)$

$= x^2 - 10x - 4x + 40$

$= x^2 - 14x + 40$ ✓

Final answer:

$$\boxed{-(x-4)(x-10)}$$

Ex: $x^4 + 3x^3 - 180x^2$

$= x^2(x^2 + 3x - 180)$ ← (−) signs are opposite want difference of $3x$ for middle term

$= \boxed{x^2(x + 15)(x - 12)}$

180
1 · 180
10 · 18
9 · 20
4 · 45
30 · 6
36 · 5
2 · 90
3 · 60
15 · 12

180
10 · 18
2 · 5 · 2 · 9
2 · 5 · 2 · 3 · 3

Check: $(x+15)(x-12)$

$= x^2 - 12x + 15x - 180$

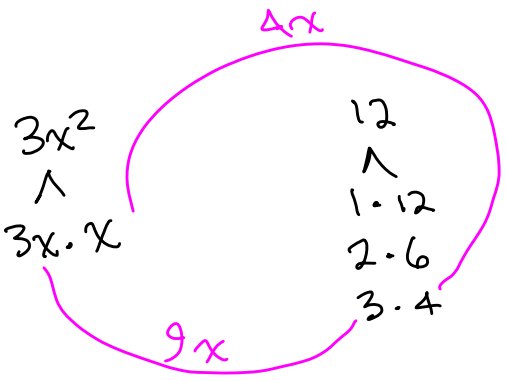
$= x^2 + 3x - 180$ ✓

$\begin{array}{r} 1 \\ 15 \\ 12 \\ \hline 30 \\ 15 \\ \hline 180 \end{array}$ ✓

Factor.

Ex: $3x^2 - 13x + 12$ ← (+) same signs sum of $13x$ for middle term

$\boxed{(3x - 4)(x - 3)}$



Check: $3x^2 - 9x - 4x + 12$

$= 3x^2 - 13x + 12$ ✓

Also correct: $\boxed{(x-3)(3x-4)}$

Ex: $6x^2 - x - 15$ (− signs are opposite want a difference of 1x)

$(2x + 3)(3x - 5)$

$6x^2$
$x \cdot 6x$
$2x \cdot 3x$

15
$1 \cdot 15$
$3 \cdot 5$

$9x$
$10x$

Check: $6x^2 - 10x + 9x - 15$
$= 6x^2 - x - 15$ ✓

Also correct: $(3x-5)(2x+3)$

Ex: $10x^2y + 49xy^2 - 5y^3$ (− signs are opposite want a difference of 49)

$= y(10x^2 + 49xy - 5y^2)$
$= y(x + 5y)(10x - y)$

$10x^2$
$x \cdot 10x$
$2x \cdot 5x$

$5y^2$
$1y \cdot 5y$

$50xy$
$1xy$

Check: $(x+5y)(10x-y)$
$= 10x^2 - xy + 50xy - 5y^2$
$= 10x^2 + 49xy - 5y^2$ ✓

5.4: Difference of Two Squares and Perfect Square Trinomials

Recall: $(a+b)(a-b) = a^2 - ab + ab - b^2$
$= a^2 - b^2$

Difference of 2 squares Factorization

$$a^2 - b^2 = (a+b)(a-b)$$

Ex: Factor.

$$x^2 - 64$$
$$= x^2 - 8^2$$
$$= \boxed{(x+8)(x-8)}$$

Also correct: $\boxed{(x-8)(x+8)}$

Check: $(x+8)(x-8)$
$= x^2 - 8x + 8x - 64$
$= x^2 - 64$ ✓

Ex: $4x^2 - 25$

Think of $a = 2x$, $b = 5$

$= (2x)^2 - (5)^2$
$= \boxed{(2x+5)(2x-5)}$ also correct: $(2x-5)(2x+5)$

Check: $4x^2 - 10x + 10x - 25$
$= 4x^2 - 25$ ✓

Ex: $100y^2 - 81z^2$

$\boxed{(10y - 9z)(10y + 9z)}$

Ex: $9x^3 - 16x$
$= x(9x^2 - 16)$
$= \boxed{x(3x+4)(3x-4)}$

Ex: $x^2 + 4$

Try $(x+2)(x+2)$

Check: $x^2 + 2x + 2x + 4$

$= x^2 + 4x + 4$ No!

Try $(x+2)(x-2)$

Check: $x^2 - 2x + 2x - 4$

$= x^2 - 4$ No!

$x^2 + 4$

$= x^2 + 0x + 4$ ← (+) same signs for middle term

sum of $0x$

Impossible!

This polynomial is prime. (cannot be factored)

Important fact:

$x^2 +$ positive is always prime.

$a^2 + b^2$ is prime.

Ex: $x^5 - x$

$= x(x^4 - 1)$

$= x((x^2)^2 - (1)^2)$

$= x(x^2 + 1)(x^2 - 1)$

$= x(x^2 + 1)(x + 1)(x - 1)$

Ex.: $x^2 + 18x + 81$ (+) same signs, want sum of 18x

$= (x+9)(x+9)$

x^2 ^ $x \cdot x$

81 ^ 9·9 ^ 1·81 ^ 3·27

Check: $x^2 + 9x + 9x + 81$

$= x^2 + 18x + 81$ ✓

Correct Final answer: $\boxed{(x+9)^2}$

Perfect Square Trinomials:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Ex.: $16x^2 - 56xy + 49y^2$

Try $(4x - 7y)^2$

Check: $(4x-7y)(4x-7y)$

$= 16x^2 - 28xy - 28xy + 49y^2$

$= 16x^2 - 56xy + 49y^2$ It works!

28 + 28 = 56

Answer: $\boxed{(4x-7y)^2}$

If the middle term didn't turn out correct, then start over and choose numbers just as in other trinomials (in that case, it's not a perfect square trinomial). See 5:30 notes for an example of this.