Note Title 11/4/2015

Back to 5.3 temporarily:

Ex: Factor.

$$4x^2 - 7x - 15$$

If trial-and-error factoring isn't clicking for you, try the **Grouping Method**:

To factor $ax^2 + bx + c$ by the grouping method, look for two numbers whose product is ac and whose sum (incorporating signs) is b.

$4x^2 - 7x - 15$. (-) signs are opposite want a difference of 7

$a = 4$
$c = 15$
Product: $ac = 4(15) = 60$

60
1 · 60
2 · 30
3 · 20
4 · 15
5 · 12
6 · 10

$4x^2 + 5x - 12x - 15$

$(4x^2 + 5x) + (-12x - 15)$

$x(4x+5) - 3(4x+5)$

$\boxed{(4x+5)(x-3)}$

Check: $4x^2 - 12x + 5x - 15$

$4x^2 - 7x - 15$ ✓

By Trial-and Error:

$4x^2 - 7x - 15$

$\boxed{(4x + 5)(x - 3)}$

$4x^2$
$4x \cdot x$
$2x \cdot 2x$

15
1 · 15
3 · 5

5x
12x

5.5: Sum and difference of 2 cubes

Sum and Difference of 2 cubes Factorization:

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$
$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

Mnemonic Devic (for remembering the sign)

M	Match	S	Same
O	Opposit	O	Opposite
P	Positive	A	Always
		P	Positive

Perfect Cubes:

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1000$

Ex: $y^3 - 27$

use $a^3-b^3 = (a-b)(a^2+ab+b^2)$ with $a=y$, $b=3$

$= y^3 - 3^3$

$= (y-3)(y^2+3y+3^2)$

$= \boxed{(y-3)(y^2+3y+9)}$

Check:

$(y-3)(y^2+3y+9)$

$= y^3 + 3y^2 + 9y - 3y^2 - 9y - 27$

$= y^3 - 27$

Ex: Factor.

$$125x^3 + 64$$

$$= (5x)^3 + (4)^3$$

$$= (5x + 4)\left((5x)^2 - (5x)(4) + 4^2\right)$$

$$= \boxed{(5x+4)(25x^2 - 20x + 16)}$$

Ex: Factor.

$$27w^3 - 8c^3$$

$$= (3w)^3 - (2c)^3$$

$$= (3w-2c)\left((3w)^2 + 3w(2c) + (2c)^2\right)$$

$$= \boxed{(3w-2c)(9w^2 + 6wc + 4c^2)}$$

Ex: $128x^4y + 250y^4x$

$$= 2xy\left(64x^3 + 125y^3\right)$$

$$= 2xy\left[\ (4x)^3 + (5y)^3\ \right]$$

$$= 2xy\left[\ (4x+5y)\left((4x)^2 - (4x)(5y) + (5y)^2\right)\ \right]$$

$$= \boxed{2xy(4x+5y)(16x^2 - 20xy + 25y^2)}$$

Check: $(4x+5y)(16x^2 - 20xy + 25y^2)$

$$= 64x^3 - 80x^2y + 100xy^2 + 80x^2y - 100xy^2 + 125y^3$$

$$= 64x^3 + 125y^3 \ \checkmark$$

Multiply by $2xy$: $2xy(64x^3 + 125y^3) = 128x^4y + 250xy^4$ ✓ ok

★ Very important: work all the problems from 5.6: Factoring – A general review

5.7: Solving Quadratic Equations by Factoring

A quadratic equation (in x) is an equation that can be written in the form $ax^2+bx+c=0$, where a, b, and c are real numbers and $a \neq 0$.

$ax^2 + bx + c = 0$ is standard form for a quadratic eqn.

ax^2: quadratic term; bx: linear term; c: constant term

Zero Product Property of numbers (or the Zero Product Theorem)

If the product of two numbers is 0, then at least one of the numbers must be 0.

In other words, if $AB=0$, then $A=0$ or $B=0$.

Steps for solving a quadratic eqn:

1) Write it in standard form $ax^2+bx+c=0$.
2) Factor the nonzero side.
3) Set each factor equal to 0.
4) Solve the resulting linear eqns.
5) Write the solution set.

Note:

Expressions get simplified (or factored)

Equations get solved.

Ex: Solve.

$$x^2 - 24 = 10x$$

(−) signs are opposite

$$x^2 - 10x - 24 = 0$$

difference of 10

$$(x + 2)(x - 12) = 0$$

$x + 2 = 0$ OR $x - 12 = 0$

$-2 \quad -2$ $+12 \quad +12$

$x = -2$ | $x = 12$

Solution Set: $\{-2, 12\}$

Also correct: $\{12, -2\}$

Check: $(x+2)(x-12)$

$= x^2 - 12x + 2x - 24$

$= x^2 - 10x - 24$

Check Solutions in Original Eqn:

$x^2 - 24 = 10x$

$x = -2$: $(-2)^2 - 24 = 10(-2)$

$4 - 24 = -20$

$-20 = -20$ ✓

$x = 12$: $12^2 - 24 = 10(12)$

$144 - 24 = 120$

$120 = 120$ ✓

Ex: Solve.

$$2x^2 - 14x = -24$$

$$2x^2 - 14x + 24 = 0$$

$$2(x^2 - 7x + 12) = 0$$

$$2(x - 4)(x - 3) = 0$$

$2 = 0$ OR $x - 4 = 0$ OR $x - 3 = 0$

never true | $x = 4$ | $x = 3$

Solution Set: $\{4, 3\}$

Ex: $x^3 = 9x$

$x^3 - 9x = 0$

$x(x^2 - 9) = 0$

$x(x+3)(x-3) = 0$

$x = 0$ or $x + 3 = 0$ or $x - 3 = 0$

$-3 \quad -3$

$x = -3$ | $x = 3$

Solution Set: $\{0, 3, -3\}$

Sometimes written $\{0, \pm 3\}$

± 3: "plus or minus 3"