5.8: Applications of Quadratic Equations (cont'd)

Note Title 11/16/2015

Ex: The height of a triangle is 4 ft less than three times its base. The area of the triangle is 42 square feet. Find the base and height.

base: x

height: $3x-4$

height $\xrightarrow{\text{compared to}}$ base x

$$\text{Area of triangle} = \frac{1}{2}(\text{base})(\text{height})$$

$$42 = \frac{1}{2}(x)(3x-4)$$

Multiply both sides by 2: $(2)(42) = 2\left(\frac{1}{2}x\right)(3x-4)$

$$84 = x(3x-4)$$

$$84 = 3x^2 - 4x$$

$$0 = 3x^2 - 4x - 84$$

$$0 = (3x+14)(x-6)$$

$3x+14=0$ OR $x-6=0$

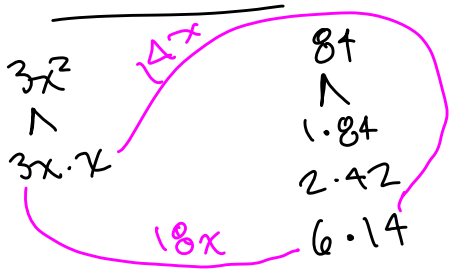
$3x = -14$ $\quad x = 6$

$\frac{3x}{3} = \frac{-14}{3}$

$x = -\frac{14}{3}$

$x = -4\frac{2}{3}$

Scratchwork



84

$2 \cdot 42$

$2 \cdot 6 \cdot 7$

$2 \cdot 2 \cdot 3 \cdot 7$

14×6

84

Check:

$(3x+14)(x-6)$

$= 3x^2 - 18x + 14x - 84$

$= 3x^2 - 4x - 84$ ✓

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Previous example cont'd:

$x = -4\frac{2}{3}$ | $x = 6$

A negative number does not make sense for a dimension. Throw out $x = -\frac{14}{3}$

base: $x = 6$ ft

height: $3x - 4 = 3(6) - 4$
$= 18 - 4$
$= 14$ ft

The base is 6 ft and the height is 14 ft.

Check our answer:

3 times base: $3(6\text{ ft}) = 18\text{ ft}$

4 ft less: 14 ft ✓ok

$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$
$= \frac{1}{2}(6\text{ ft})(14\text{ ft})$
$= 3\text{ ft}(14\text{ ft}) = 42\text{ ft}^2$ ✓ok

$14 \times 3 = 42$

Ex: Find two consecutive integers whose product is 27 more than 5 times the larger integer.

1st integer: x

2nd integer: $x+1$

$$\text{Product} = 5(\text{larger integer}) + 27$$

$$x(x+1) = 5(x+1) + 27$$

$$x^2 + x = 5x + 5 + 27$$

$$x^2 + x = 5x + 32$$

$$x^2 + x - 5x - 32 = 0$$

$$x^2 - 4x - 32 = 0$$

$$(x+4)(x-8) = 0$$

$x+4=0$	$x-8=0$
$x=-4$	$x=8$

These are two different possibilities for the first integer.

$x=8$ 1st integer: $x=8$

2nd integer: $x+1 = 8+1 = 9$

This gives us an integer pair 8, 9.

$x=-4$ 1st integer: $x=-4$

2nd integer: $x+1 = -4+1 = -3$

This gives us an integer pair $-4, -3$.

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The pair of integers is either 8, 9 or $-4, -3$.

Check answer:

Check 8, 9 first | Are they consecutive integers? Yes

Product: $8(9) = 72$ ← equal ✓

5 times larger: $5(9) = 45$

27 more: $27 + 45 = 72$ ←

$$\begin{array}{r} 1 \\ 45 \\ +27 \\ \hline 72 \end{array}$$

Now check $-4, -3$ | Are they consecutive integers? Yes ✓

Product: $-4(-3) = 12$ ← equal ✓

5 times larger: $5(-3) = -15$

27 more: $-15 + 27 = 12$ ←

$$\begin{array}{r} 27 \\ -15 \\ \hline 12 \end{array}$$

Ex: Find two consecutive odd integers whose product is one less than six times their sum.

1st integer: x

2nd integer: $x+2$

$$\text{Product} = 6(\text{sum}) - 1$$

$$x(x+2) = 6(x+x+2) - 1$$

$$x^2 + 2x = 6x + 6x + 12 - 1$$

$$x^2 + 2x = 12x + 11$$

$$x^2 + 2x - 12x - 11 = 0$$

$$x^2 - 10x - 11 = 0$$

$$(x - 11)(x + 1) = 0$$

$x - 11 = 0$ | $x + 1 = 0$

$x = 11$ | $x = -1$

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Previous example cont'd:

$x = 11$ 1st integer: $x = 11$

2nd integer: $x+2 = 11+2 = 13$

$x = -1$ 1st integer: $x = -1$

2nd integer: $x+2 = -1+2 = 1$

The pair of integers is either 11, 13 or $-1, 1$.

Check:

11, 13 Consecutive odd integers? Yes

Product: $11(13) = 143$

Sum: $11+13 = 24$

6 times sum: $6(24) = 144$

1 less: 143

equal ✓

$$\begin{array}{r} 13 \\ \times\ 11 \\ \hline 13 \\ 13 \\ \hline 143 \end{array}$$

$$\begin{array}{r} {\scriptstyle 2}24 \\ \times\ 6 \\ \hline 144 \end{array}$$

$-1, 1$ consecutive odd integers? yes

Product: $-1(1) = -1$

Sum: $-1+1 = 0$

6 times sum: $6(0) = 0$

1 less: -1

equal ✓

Chapter 6: Rational Expressions

6.1: Simplifying Rational Expressions

Recall: **Rational Number:** can be written as the ratio (fraction, quotient) of 2 integers.

In other words, a rational number can be written as $\frac{a}{b}$, where $b \neq 0$, and a, b are integers.

Examples of rational numbers:

$$\frac{2}{3}, \quad -\frac{3}{5}, \quad \frac{5}{3}, \quad 2\frac{1}{4}, \quad 2, \quad -3, \quad 0, \quad 3.14$$

$2\frac{1}{4} = \frac{9}{4}$, $2 = \frac{2}{1}$, $-3 = \frac{-3}{1}$, $0 = \frac{0}{1}$, $3.14 = \frac{314}{100}$

Irrational numbers (not rational): $\sqrt{2}, \sqrt{3}, \pi, e$

Rational Expressions: can be written as a ratio of two polynomials (with denominator not a constant)

Examples of Rational Expressions:

$$\frac{2x-3}{x+5}, \quad \frac{x^2+2x-1}{3x-2}, \quad \frac{1}{x^2-9}$$

To simplify rational expressions, we "cancel out" (divide out) factors that appear in both the numerator and denominator.

Ex: $\dfrac{24x^4y^3}{32x^5y}$ Simplify.

$= \boxed{\dfrac{3y^2}{4x}}$

Ex: Simplify.

$\dfrac{x^2+8x+15}{x^2+5x+6} = \dfrac{(x+3)(x+5)}{(x+2)(x+3)}$

$= \boxed{\dfrac{x+5}{x+2}}$

Ex: Simplify.

$\dfrac{x-3}{x^2-7x+12} = \dfrac{x-3}{(x-3)(x-4)} = \boxed{\dfrac{1}{x-4}}$