

6.4: Solving Equations with Rational Expressions

Note Title

11/23/2015

Ex. $\frac{x}{5} + \frac{1}{5} = \frac{4}{5}$ solve.

what values of x make the eqn true? 3

Solve by inspection: $x=3$

Check it: $x=3 \Rightarrow \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

$$\frac{5}{5} = \frac{5}{5}$$

$$1 = 1 \quad \checkmark \text{ True}$$

Sol'n Set: $\boxed{\{3\}}$

Ex. Solve.

$$\frac{x}{8} = \frac{7}{8}$$

By inspection: $x=7$

Sol'n Set: $\boxed{\{7\}}$

Check: $\frac{7}{8} = \frac{7}{8} \quad \checkmark \text{ True}$

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Note: If 2 fractions are equal, and they have the same denominator, then their numerators must be equal.

Ex:

Solve.

$$\frac{x}{5} - \frac{4x}{3} = \frac{1}{15}$$

LCD: 15

$$\frac{x}{5} \left(\frac{3}{3}\right) + \frac{-4x}{3} \left(\frac{5}{5}\right) = \frac{1}{15}$$

$$\frac{3x}{15} + \frac{-20x}{15} = \frac{1}{15}$$

$$\frac{-17x}{15} = \frac{1}{15}$$

$$\left(\frac{15}{1}\right) \frac{-17x}{15} = \frac{1}{15} \left(\frac{15}{1}\right)$$

$$-17x = 1$$

$$\frac{-17x}{-17} = \frac{1}{-17}$$

$$x = -\frac{1}{17}$$

Sol'n Set:

$$\left\{-\frac{1}{17}\right\}$$

Ex:

Solve.

$$\frac{2x-1}{2} - \frac{4x+3}{3} = \frac{x+5}{6}$$

LCD: 6

$$\frac{2x-1}{2} \left(\frac{3}{3}\right) + \frac{-4x-3}{3} \left(\frac{2}{2}\right) = \frac{x+5}{6}$$

$$\frac{6x-3}{6} + \frac{-8x-6}{6} = \frac{x+5}{6}$$

$$\left(\frac{6}{1}\right) \frac{6x-3-8x-6}{6} = \frac{x+5}{6} \left(\frac{6}{1}\right)$$

$$6x-3-8x-6 = x+5$$

See next
page

Previous example
cont'd:

$$6x - 3 - 8x - 6 = x + 5$$

$$-2x - 9 = x + 5$$

$$-2x - x = 5 + 9$$

$$-3x = 14$$

$$\frac{-3x}{-3} = \frac{14}{-3}$$

$$x = -\frac{14}{3}$$

$$\boxed{\left\{-\frac{14}{3}\right\}} \text{ sol'n set}$$

Ex: Solve.

$$\frac{1}{3x} - \frac{1}{6} = \frac{5}{x}$$

LCD: $6x$

Restriction: $x \neq 0$

$$\frac{1}{3x} \left(\frac{2}{2} \right) + \frac{-1}{6} \left(\frac{x}{x} \right) = \frac{5}{x} \left(\frac{6}{6} \right)$$

$$\frac{2}{6x} + \frac{-x}{6x} = \frac{30}{6x}$$

$$\left(\frac{6x}{1} \right) \frac{2-x}{6x} = \frac{30}{6x} \left(\frac{6x}{1} \right)$$

$$2 - x = 30$$

$$-x = 30 - 2$$

$$-x = 28$$

$$x = -28$$

$$\frac{-x}{-1} = \frac{28}{-1}$$

$$\text{Sol'n set: } \boxed{\{-28\}}$$

Ex.

Solve.

$$\frac{x}{x+16} = \frac{2}{x-2}$$

$$\text{LCD: } (x+16)(x-2)$$

$$\text{Restrictions: } x \neq 2 \\ x \neq -16$$

$$\frac{x}{x+16} \left(\frac{x-2}{x-2} \right) = \frac{2}{x-2} \left(\frac{x+16}{x+16} \right)$$

$$\frac{x^2 - 2x}{(x+16)(x-2)} = \frac{2x + 32}{(x-2)(x+16)}$$

$$\frac{\cancel{(x-2)} \cancel{(x+16)}}{1} \frac{x^2 - 2x}{\cancel{(x+16)} \cancel{(x-2)}} = \frac{2x + 32}{\cancel{(x-2)} \cancel{(x+16)}} \frac{\cancel{(x-2)} \cancel{(x+16)}}{1}$$

$$x^2 - 2x = 2x + 32$$

$$x^2 - 2x - 2x - 32 = 0$$

$$x^2 - 4x - 32 = 0$$

$$(x-8)(x+4) = 0$$

$$x-8=0 \quad \text{or} \quad x+4=0$$

$$x=8$$

$$x=-4$$

$$\text{Sol'n Set: } \boxed{\{8, -4\}}$$

Ex: $\frac{2x}{x-3} - \frac{1}{2} = \frac{6}{x-3}$

Solve.

LCD: $2(x-3)$

$$\frac{2x}{x-3} \left(\frac{2}{2}\right) + \frac{-1}{2} \left(\frac{x-3}{x-3}\right) = \frac{6}{x-3} \left(\frac{2}{2}\right)$$

Restrictions: $x \neq 3$

$$\frac{4x - x + 3}{2(x-3)} = \frac{12}{2(x-3)}$$

$$\cancel{\frac{(2(x-3))}{1}} \frac{3x+3}{\cancel{2(x-3)}} = \frac{12}{\cancel{2(x-3)}} \frac{\cancel{(2)(x-3)}}{1}$$

$$3x + 3 = 12$$

$$3x = 9$$

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

Check it: Put $x=3$ into original eqn: $\frac{2x}{x-3} - \frac{1}{2} = \frac{6}{x-3}$

$$x=3 \Rightarrow \frac{2(3)}{3-3} - \frac{1}{2} = \frac{6}{3-3}$$

Division by 0
is undefined!!

Throw out $x=3$.

$$\frac{6}{0} - \frac{1}{2} = \frac{6}{0}$$

No Solution



Important:- When you solve an equation that has variables in the denominator, you MUST CHECK your solutions. If a solution gives you a zero denominator when substituted into the original eqn, throw it out.

Extraneous Solution (not a solution at all)

It's a value generated by a correct solution process that does not check in the original equation.

Ex: $\frac{x}{x-2} + 3 = \frac{10}{x-2}$

Restriction: $x \neq 2$

$$\frac{x}{x-2} + \frac{3}{1} \left(\frac{x-2}{x-2} \right) = \frac{10}{x-2}$$

LCD: $x-2$

$$\left(\frac{\cancel{x-2}}{1} \right) \frac{x+3x-6}{\cancel{x-2}} = \frac{10}{\cancel{x-2}} \left(\frac{\cancel{x-2}}{1} \right)$$

$$x+3x-6 = 10$$

$$4x-6 = 10$$

$$4x = 16$$

$$\frac{4x}{4} = \frac{16}{4}$$

$$x = 4$$

Sol'n Set: $\boxed{\{4\}}$

Does this cause zero denominators in original eqn? No

Chapter 7: Solving Linear Systems

7.1: Solving Linear Systems by Graphing

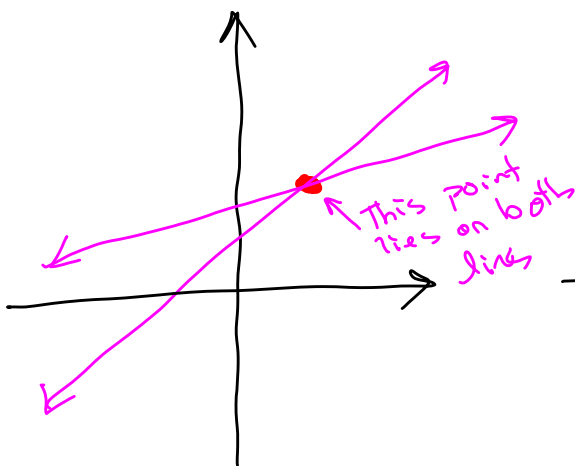
System of equations: group of 2 or more equations

Solution to a system of eqns: A set of values for the variables that make all the equations true.

For us: we will solve systems of 2 linear equations in 2 variables.

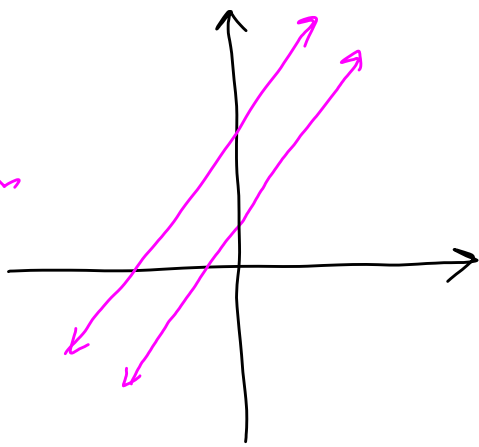
Ex:
$$\begin{cases} 3x - 4y = 12 \\ x + 8y = -17 \end{cases}$$
 $\swarrow \searrow$ graphs of these eqns are lines

3 possible scenarios for a system of 2 linear eqns in 2 variables



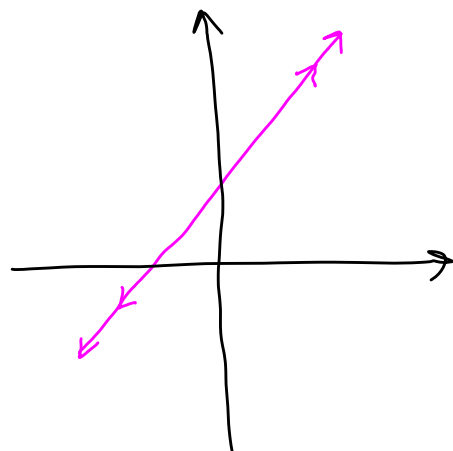
Independent System

One solution
Lines intersect at a single point.



Inconsistent System

No solution
Lines are parallel



Dependent System

Infinitely many solutions
Lines are the same

Question on Test #5

For an inconsistent system, a dependent system, or an independent system:

- 1) State the number of solutions.
- 2) Describe the graph in words (lines are parallel, lines are the same, lines intersect at a single point).
- 3) Illustrate with an example graph.

Example: Is the ordered pair $(9, -2)$ a solution of this system?

$$\begin{aligned}2x + 5y &= 8 \\ 3x - 2y &= 23\end{aligned}$$

Put $x=9, y=-2$ into $2x+5y=8$:

$$2(9) + 5(-2) = 8$$

$$18 - 10 = 8$$

$$8 = 8 \quad \checkmark \text{ true}$$

Put $x=9, y=-2$ into $3x-2y=23$.

$$3(9) - 2(-2) = 23$$

$$27 + 4 = 23$$

$$31 = 23 \quad \text{False}$$

No, $(9, -2)$ is not a solution to the system.

7.1: To solve a system by graphing, graph both lines and look for the point that lies on both lines (if there is one)

We will skip this section. You can do the HW on 7.1 to get a bonus or replace a low/missing HW grade.

7.2: Solving Systems by the Substitution Method

Example: Solve by substitution.

$$x - y = 9$$

$$2x - 11y = -18$$

① Solve one eqn (your choice) for one variable (your choice).

Solve $x - y = 9$ for x :

$$x = 9 + y$$

② Substitute this expression into the other eqn.

Here, substitute $x = 9 + y$ into $2x - 11y = -18$.

$$2(9 + y) - 11y = -18$$

③ Solve for the remaining variable.

Here, solve for y : $2(9 + y) - 11y = -18$

$$18 + 2y - 11y = -18$$

$$18 - 9y = -18$$

$$-9y = -36$$

$$\frac{-9y}{-9} = \frac{-36}{-9}$$

$$y = 4$$

④ Put this value into either equation and solve.

Put $y = 4$ into $x - y = 9$

$$x - 4 = 9$$

$$x = 13$$

Solution is $(13, 4)$.

See next page

⑤ check your solution.

$$x - y = 9$$

$$x = 13, y = 4 \Rightarrow 13 - 4 = 9$$

$$9 = 9 \checkmark$$

$$2x - 11y = -18$$

$$x = 13, y = 4 \Rightarrow 2(13) - 11(4) = -18$$

$$26 - 44 = -18$$

$$-18 = -18 \checkmark$$