7.2: Solving Systems by substitution (Cont'd)

Note Title 11/30/2015

Ex: Solve by substitution.

$$\begin{cases} -4x + 3y = 19 \\ -6x - 5y = 0 \end{cases}$$

1) Solve $-6x - 5y = 0$ for y:

$$-5y = 6x$$

$$\frac{-5y}{-5} = \frac{6x}{-5}$$

$$y = -\frac{6}{5}x$$

2) Put $y = -\frac{6}{5}x$ into $-4x + 3y = 19$:

$$-4x + 3\left(-\frac{6}{5}x\right) = 19$$

$$-4x - \frac{18}{5}x = 19$$

$$\left(\frac{5}{5}\right)\left(\frac{-4x}{1}\right) - \frac{18}{5}x = 19$$

$$-\frac{20}{5}x - \frac{18}{5}x = 19$$

$$-\frac{38}{5}x = 19$$

$$\left(-\frac{5}{38}\right)\left(-\frac{38}{5}x\right) = \frac{19}{1}\left(\frac{-5}{38}\right)$$

$$x = -\frac{5}{2}$$

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Put $x = -\frac{5}{2}$ into $-4x + 3y = 19$:

$$-4\left(-\frac{5}{2}\right) + 3y = 19$$

$$\frac{20}{2} + 3y = 19$$

$$10 + 3y = 19$$

$$3y = 9$$

$$\frac{3y}{3} = \frac{9}{3}$$

$$y = 3$$

Solution: $\boxed{\left(-\frac{5}{2}, 3\right)}$ or $\left\{\left(-\frac{5}{2}, 3\right)\right\}$

Check your solution:

$-4x + 3y = 19$

$x = -\frac{5}{2}, y = 3 \Rightarrow -4\left(\frac{-5}{2}\right) + 3(3) = 19$

$$\frac{20}{2} + 9 = 19$$

$$10 + 9 = 19$$

$$19 = 19 \checkmark$$

$-6x - 5y = 0$

$$-6\left(-\frac{5}{2}\right) - 5(3) = 0$$

$$\frac{30}{2} - 15 = 0$$

$$15 - 15 = 0$$

$$0 = 0 \checkmark$$

7.3: The Elimination method (The addition method)

(for solving linear systems)

Steps to solving by elimination:

1) Multiply both sides of one or both equations by a strategic number, chosen so that one variable will be eliminated when the equations are added.

2) Add the equations.

3) Solve for the remaining variable.

4) Find the value of the other variable, by
   - (a) putting the known value into either of the original eqns, as in the substitution method
   - or (b) Do elimination again, on the other variable.

5) Check!

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Example: Solve by elimination.

$$\begin{cases} 11x - 5y = 2 \\ 3x - y = 1 \end{cases}$$

multiply both sides of 2nd eqn by -5:

$$\begin{array}{rl} 11x - 5y = 2 & \longrightarrow \quad 11x - 5y = 2 \\ 3x - y = 1 & \xrightarrow{(-5)} \quad -15x + 5y = -5 \\ \hline \text{Add:} & \quad -4x + 0y = -3 \end{array}$$

$$-4x = -3$$

$$\frac{-4x}{-4} = \frac{-3}{-4}$$

$$x = \frac{3}{4}$$

Could put $x = \frac{3}{4}$ into either eqn and solve for y, OR, do elimination again:

$$\begin{array}{rl} 11x - 5y = 2 & \xrightarrow{(3)} \quad 33x - 15y = 6 \\ 3x - y = 1 & \xrightarrow{(-11)} \quad -33x + 11y = -11 \\ \hline \text{Add:} & \quad 0x - 4y = -5 \end{array}$$

$$-4y = -5$$

$$\frac{-4y}{-4} = \frac{-5}{-4}$$

$$y = \frac{5}{4}$$

Solution: $\left(\frac{3}{4}, \frac{5}{4}\right)$

Check on next page

Check: $11x - 5y = 2$

$11\left(\frac{3}{4}\right) - 5\left(\frac{5}{4}\right) = 2$

$\frac{33}{4} - \frac{25}{4} = 2$

$\frac{8}{4} = 2$

$2 = 2$ ✓

$3x - y = 1$

$3\left(\frac{3}{4}\right) - \frac{5}{4} = 1$

$\frac{9}{4} - \frac{5}{4} = 1$

$\frac{4}{4} = 1$

$1 = 1$ ✓

Ex: Solve.

$-5x + 7y = 2$

$-8y - 4x = -5$

Rearrange:

$-5x + 7y = 2 \xrightarrow{(4)} -20x + 28y = 8$

$-4x - 8y = -5 \xrightarrow{(-5)} 20x + 40y = 25$

Add: $0x + 68y = 33$

$68y = 33$

$y = \frac{33}{68}$

$-5x + 7y = 2 \xrightarrow{(8)} -40x + 56y = 16$

$-4x - 8y = -5 \xrightarrow{(7)} -28x - 56y = -35$

Add: $-68x + 0y = -19$

$-68x = -19$

$\frac{-68x}{-68} = \frac{-19}{-68}$

$x = \frac{19}{68}$

Solution: $\left(\frac{19}{68}, \frac{33}{68}\right)$

Check on next page:

Check:

$$-5x + 7y = 2$$
$$-8y - 4x = -5$$

$$-5x + 7y = 2$$
$$-5\left(\frac{19}{68}\right) + 7\left(\frac{33}{68}\right) = 2$$
$$-\frac{95}{68} + \frac{231}{68} = 2$$
$$\frac{136}{68} = 2$$
$$2 = 2 \checkmark$$

$$-8y - 4x = -5$$
$$-8\left(\frac{19}{68}\right) - 4\left(\frac{33}{68}\right) = -5$$
$$-8\left(\frac{33}{68}\right) - 4\left(\frac{19}{68}\right) = -5$$
$$-\frac{264}{68} - \frac{76}{68} = -5$$
$$-\frac{340}{68} = -5$$
$$-5 = -5 \checkmark$$

Ex: Solve.

$$-x + 6y = 2$$
$$3x - 18y = 5$$

$$-x + 6y = 2 \xrightarrow{(3)} -3x + 18y = 6$$
$$3x - 18y = 5 \longrightarrow 3x - 18y = 5$$

Add: $0x + 0y = 11$

$0 = 11$ False

Inconsistent System; No Solution

(lines are parallel)

Ex: Solve.

$$2x - 10y = 6$$
$$-5x + 25y = -15$$

$$2x - 10y = 6 \xrightarrow{(5)} 10x - 50y = 30$$
$$-5x + 25y = -15 \xrightarrow{(2)} -10x + 50y = -30$$

Add: $0x + 0y = 0$

$0 = 0$ True

Dependent system; infinitely many solutions

(lines are the same)

Recognizing inconsistent and dependent systems:

When solving a system by elimination or substitution,

* if both variables disappear and leave you with a false statement, the system is inconsistent and has no solution. (lines are parallel)

ex: $0 = 11$, or $2 = 5$ →

* If both variables disappear and leave you with a true statement, the system is dependent and has infinitely many solutions (lines are the same)

ex: $0 = 0$, or $2 = 2$ →

7.4: Applications of Linear Systems

Ex: Linda's age is 8 more than twice her daughter's age. The sum of their ages is 101. Find their ages.

Linda's age: x

Daughter's age: y

$$x + y = 101$$

$$\begin{pmatrix}\text{Linda's}\\ \text{age}\end{pmatrix} = 2\begin{pmatrix}\text{daughter's}\\ \text{age}\end{pmatrix} + 8$$

$$x = 2y + 8$$

Put $x = 2y + 8$ into $x + y = 101$:

$$2y + 8 + y = 101$$

$$3y + 8 = 101$$

$$3y = 93$$

$$y = \frac{93}{3} = 31$$

Put $y = 31$ into $x + y = 101$:

$$x + 31 = 101$$

$$x = 101 - 31$$

$$x = 70$$

Linda is 70 and her daughter is 31.

Ex: Diane has \$0.95 in dimes and nickels. She has a total of 11 coins. How many of each does she have?

number of dimes: d

number of nickels: n

$$d + n = 11 \quad \text{(she has 11 coins)}$$

$$\$0.10d + \$0.05n = \$0.95$$

multiply by 100:

system of 2 eqns $\begin{cases} 10d + 5n = 95 \\ d + n = 11 \end{cases}$ $\xrightarrow{(-5)}$ $-5d - 5n = -55$

$10d + 5n = 95$

Add: $5d = 40$

$\frac{5d}{5} = \frac{40}{5}$

$d = 8$

Put $d = 8$ into $d + n = 11$:

$8 + n = 11$

$n = 3$

She has 3 nickels and 8 dimes.

Solve by substitution: $10d + 5n = 95$

$d + n = 11$

Solve $d + n = 11$ for d:

$d = 11 - n$

Put $d = 11 - n$ into $10d + 5n = 95$:

$10(11 - n) + 5n = 95$

$110 - 10n + 5n = 95$

$110 - 5n = 95$

$-5n = 95 - 110$

$-5n = -15$

$n = \frac{-15}{-5} = 3$

Put $n = 3$ into $d + n = 11 \Rightarrow d + 3 = 11$

$d = 8$