

3.4: Graphing Lines Using Slope

Note Title

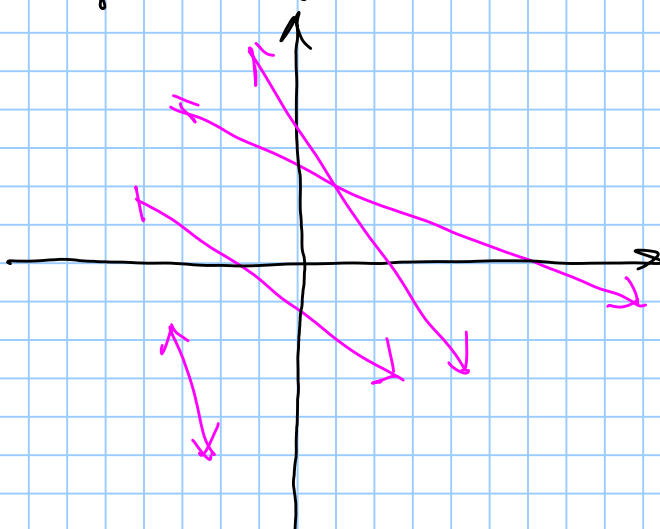
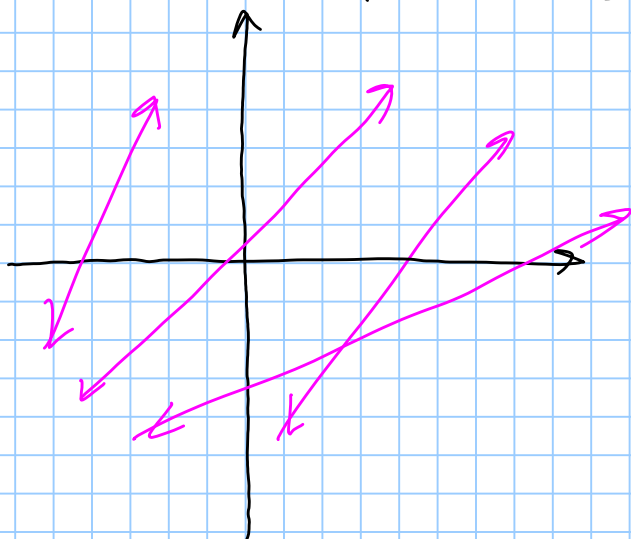
2/5/2015

Slope: measures the "steepness" of a line

Definition: $\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1)

and (x_2, y_2) are points on the line.

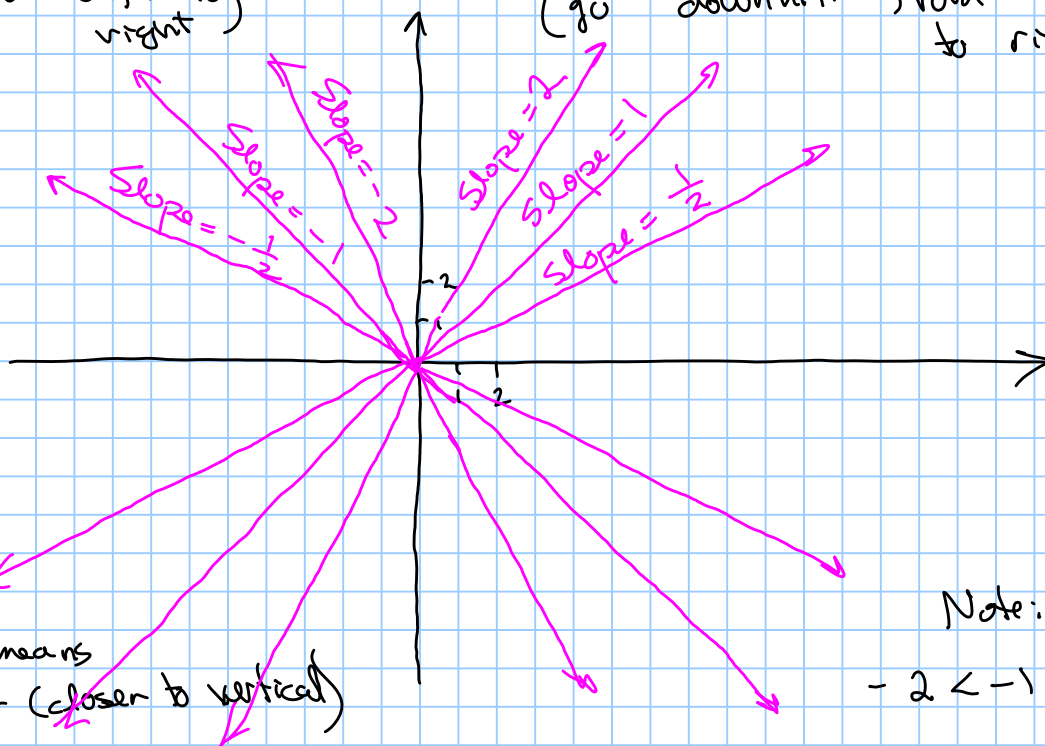
$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{Rise}}{\text{Run}}$$



Positive slopes
(go "uphill" from left to right)

Negative Slopes
(go "downhill" from left to right)

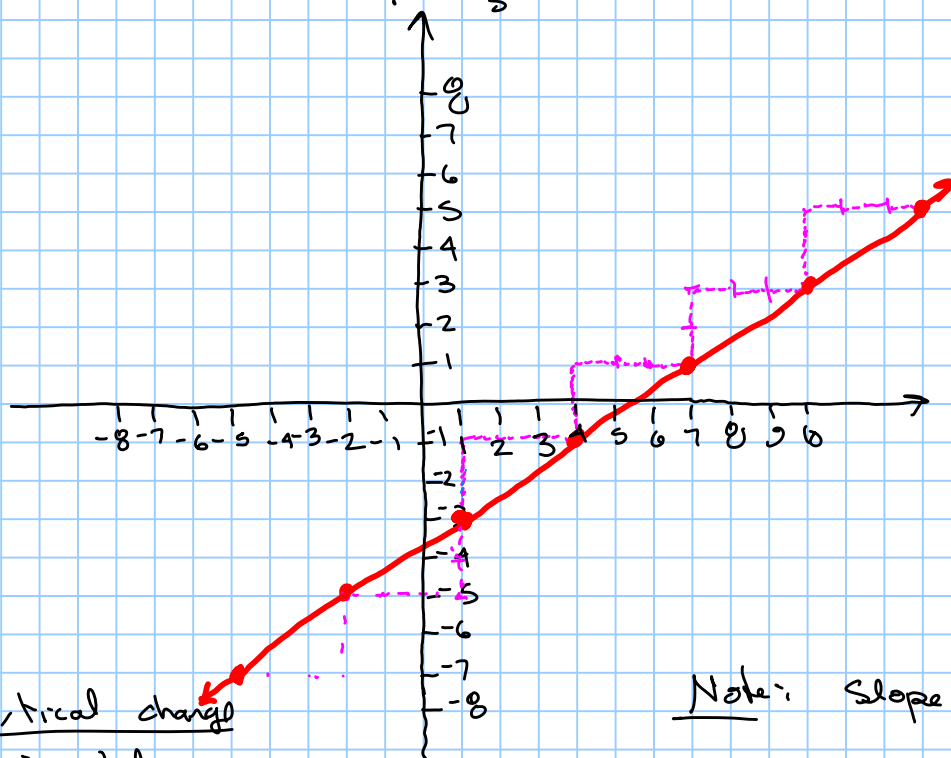
$$m = 2 = \frac{2}{1}$$
$$m = 1 = \frac{1}{1}$$



For positive slopes, a larger slope means it is steeper (closer to vertical)

Note:
 $-2 < -1 < -\frac{1}{2}$

Example: Graph the line that passes through $(1, -3)$ and has slope $\frac{2}{3}$.

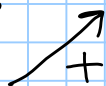


$$\begin{aligned} \text{slope} = m &= \frac{2}{3} \\ &= \frac{\text{vertical change}}{\text{horizontal change}} \\ &= \frac{\text{rise}}{\text{run}} \end{aligned}$$

Note: $\text{slope} = + \frac{2}{3} = \frac{2}{3} = \frac{-2}{-3}$

$$m = + \frac{2}{3}$$

positive slope:



vertical change: 2
horizontal change: 3

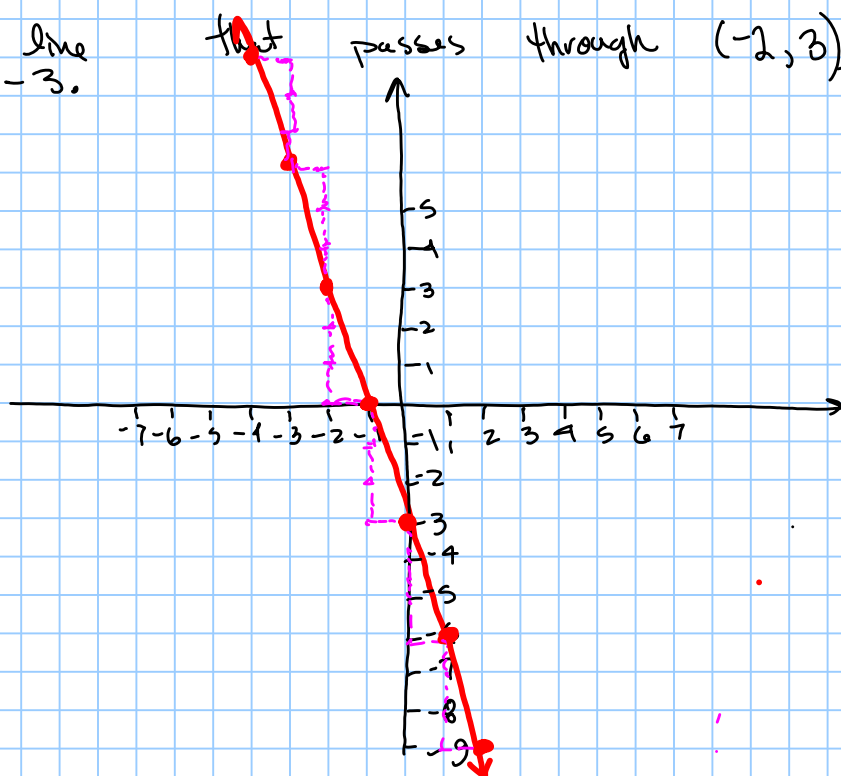
Example: Graph the line that passes through $(-2, 3)$ and has slope -3 .

$$\text{slope} = m = -3 = -\frac{3}{1} = \frac{\text{"rise"}}{\text{run}}$$

negative slope:

vertical change: 3
horizontal change: 1

Note: $m = -3 = \frac{-3}{1} = \frac{3}{-1}$



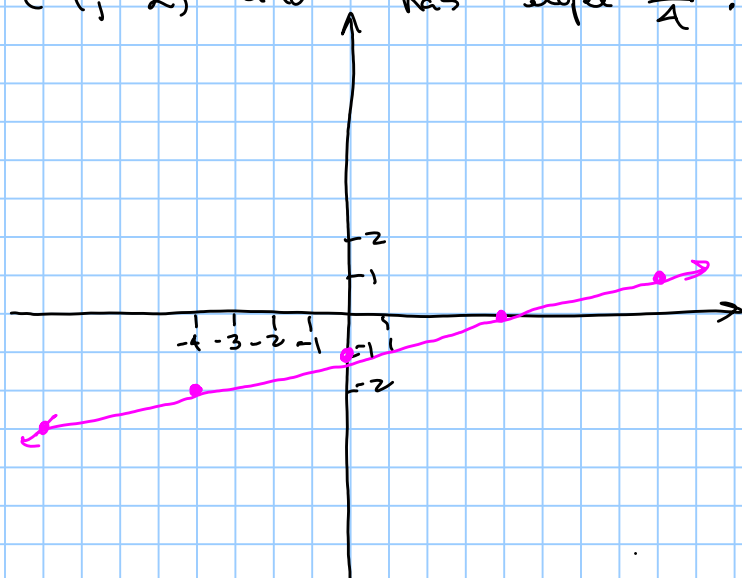
Example:

Graph the line that passes through $(-4, -2)$ and has slope $\frac{1}{4}$.

Slope: $m = +\frac{1}{4}$

Positive slope: 

vertical change: 1
horizontal change: 4



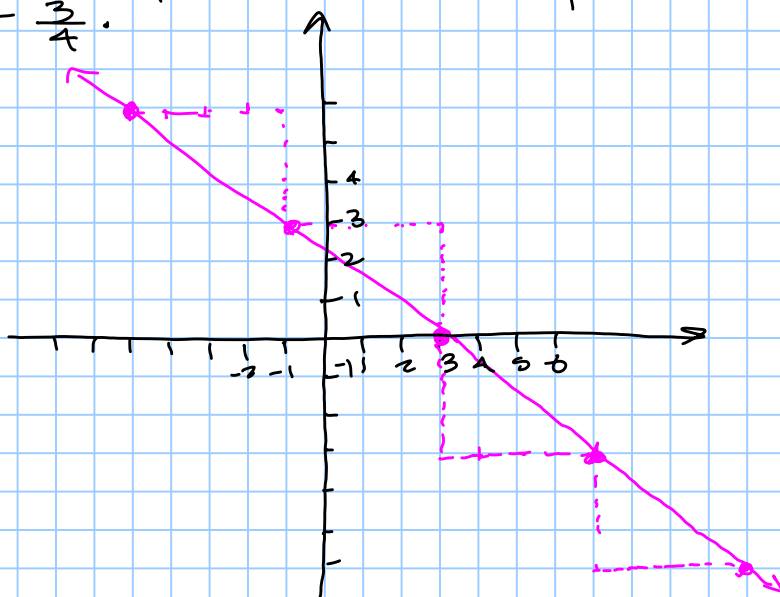
Example: Graph the line that has x-intercept 3 and slope $-\frac{3}{4}$.

Slope $m = -\frac{3}{4}$

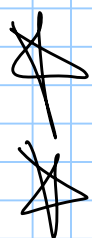
negative slope: 

vertical change: 3
horizontal change: 4

Starting point: $(3, 0)$



Slope-intercept form of a line: (Sections 3.4 and 3.5)



Slope-intercept form: $y = mx + b$, where

m is the slope and b is the y-intercept.

Example: Graph the line $y = -2x - 1$.

$$\text{Slope} = m = -2 = -\frac{2}{1}$$

Negative slope: $\searrow$

vertical change: 2

horizontal change: 1

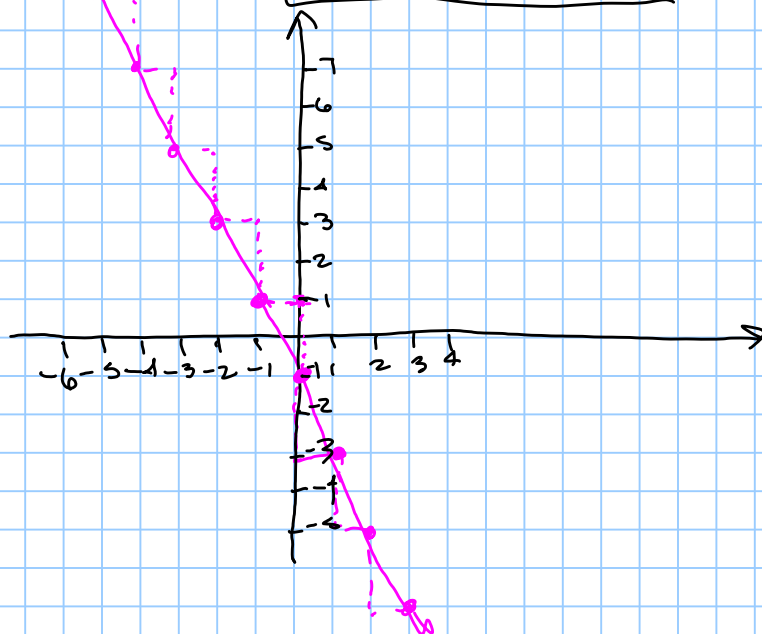
y-intercept: $b = -1$

Note: Find y-intercept by setting $x = 0$:

$$\begin{aligned} (0, -1) \quad x=0 \quad y &= -2(0) - 1 \\ y &= 0 - 1 \\ y &= -1 \end{aligned}$$

Slope-intercept form:

$$y = mx + b$$



Ex: Graph the line $y = -3x$.

Slope-intercept form: $y = mx + b$

Rewrite: $y = -3x + 0$

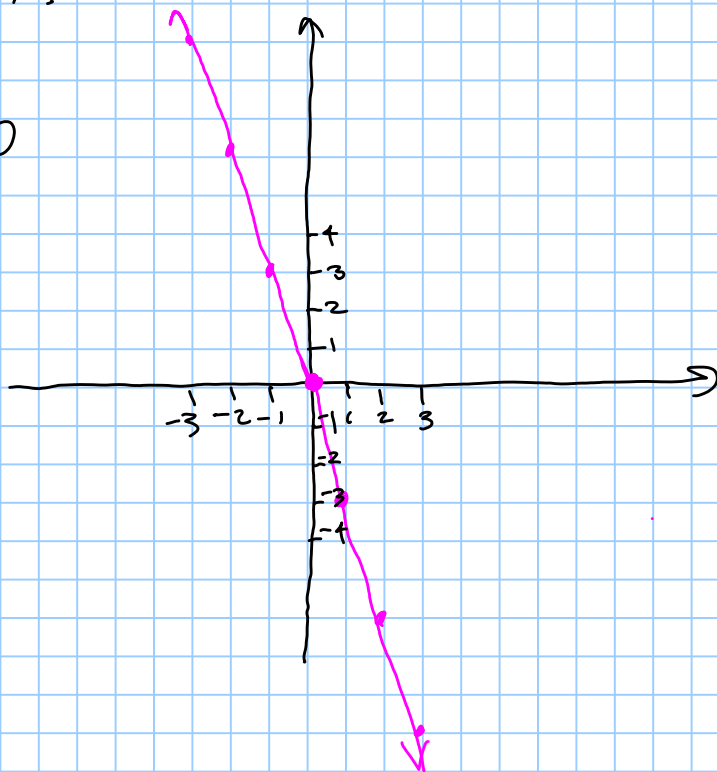
$$\text{Slope: } m = -3 = -\frac{3}{1}$$

$\searrow$

vertical change: 3

horizontal change: 1

y-intercept: $b = 0$



Example: Graph the line by writing it in slope-intercept form.

$$2x - 3y = -9$$

Want to write as $y = mx + b$.

$$-3y = -2x - 9$$

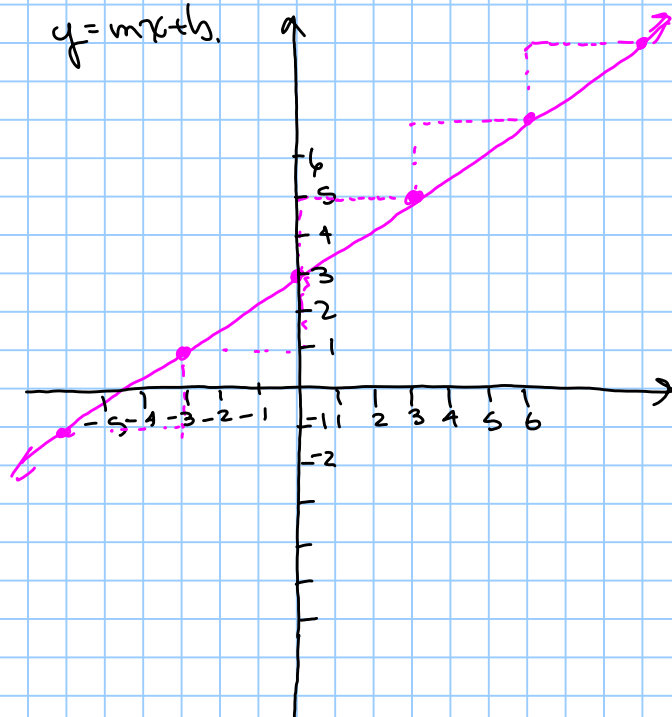
$$\frac{-3y}{-3} = \frac{-2x}{-3} - \frac{9}{-3}$$

$$\boxed{y = \frac{2}{3}x + 3}$$
 eqn in slope-intercept form

Slope: $m = +\frac{2}{3}$

Positive slope: $\nearrow +$

y-intercept: 3



3.5: Equations of Lines

Definition of Slope: $\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$, where

(x_1, y_1) and (x_2, y_2) are any two points on the line.

Example: Find the slope of the line containing the points $(-4, 1)$ and $(5, 7)$.

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Let $(x_1, y_1) = (-4, 1)$

$(x_2, y_2) = (5, 7)$

$$m = \frac{7 - 1}{5 - (-4)} = \frac{6}{5 + 4} = \frac{6}{9} = \frac{2}{3}$$

$$\boxed{\text{The slope is } \frac{2}{3}.$$

Note: we would get the same answer if we switch the order of the two points:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 7}{-4 - 5} = \frac{-6}{-9} = \frac{6}{9} = \boxed{\frac{2}{3}} \text{ same as before}$$

Ex: Find the slope of the line containing the points $(4, -6)$ and $(-1, -3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-6)}{-1 - 4} = \frac{-3 + 6}{-1 - 4} = \frac{3}{-5} = -\frac{3}{5}$$

The slope is $-\frac{3}{5}$.

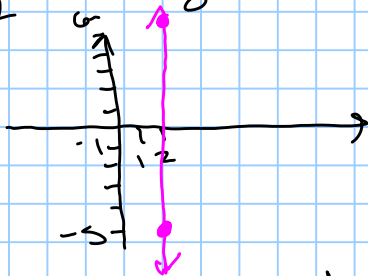
Ex: Find the slope of the line containing the points $(2, -5)$ and $(2, 6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-5)}{2 - 2} = \frac{6 + 5}{0} = \frac{11}{0}$$

Undefined.

Division by 0 is undefined.

This slope is undefined.



Vertical lines always have undefined slope

Ex: Find the slope of the line containing the points $(4, -3)$ and $(-1, -3)$.

$$m = \frac{-3 - (-3)}{-1 - 4} = \frac{-3 + 3}{-5} = \frac{0}{-5} = 0$$

The slope is 0

Horizontal lines have slope 0.

