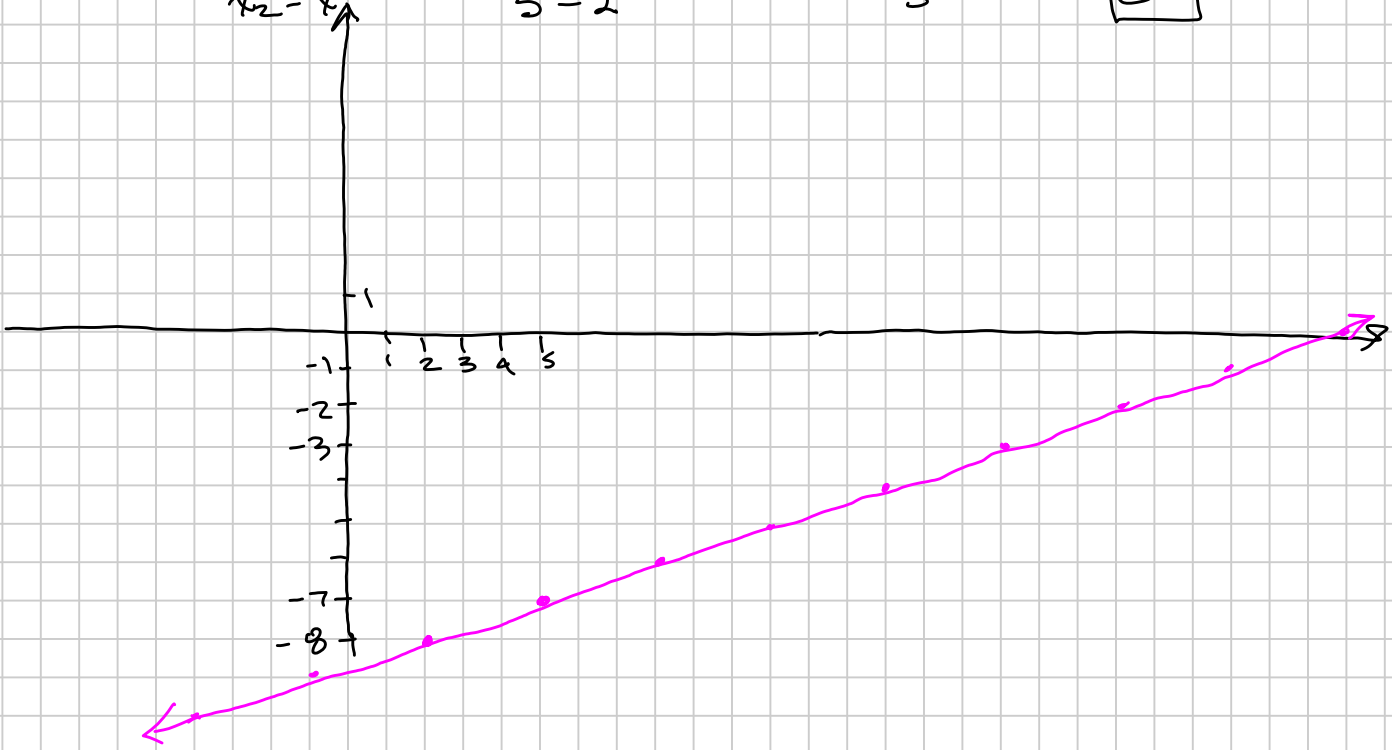
3.5: Equations of Lines (cont'd)

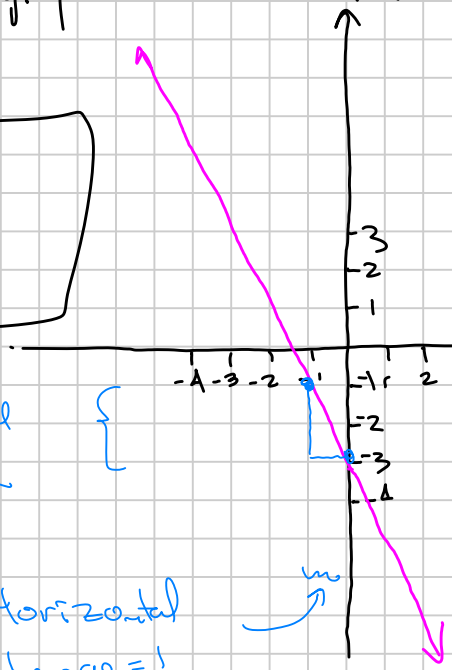
Note Title 2/5/2015

Review problem: Find the slope of the line containing the points $(2,-8)$ and $(5,-7)$. Graph it.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-(-8)}{5-2} = \frac{-7+8}{3} = \boxed{\frac{1}{3}}$$



Ex: Use the graph to write the equation of the line.



Rise = vertical change = 2

Run = Horizontal change = 1

y-intercept: $b = -3$

Is my slope positive or negative? Negative

$$\text{Slope} = m = -\frac{\text{Rise}}{\text{Run}} = -\frac{2}{1} = -2$$

Equation of Line:

$$y = -2x - 3$$

Recall: Slope-intercept form

$y = mx + b$, where m = slope, b is the y-intercept

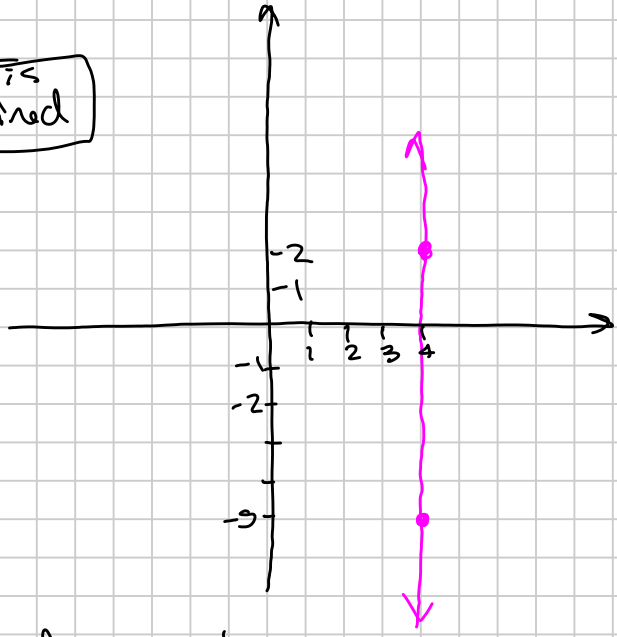
Ex: Find the slope of the line containing $(4,-5)$ and $(4,2)$.

Write the eqn of the line.

$$m = \frac{2-(-5)}{4-4} = \frac{2+5}{0} = \frac{7}{0}$$ ~~$= 0$~~

The slope is undefined

Equation of this line: $x = 4$



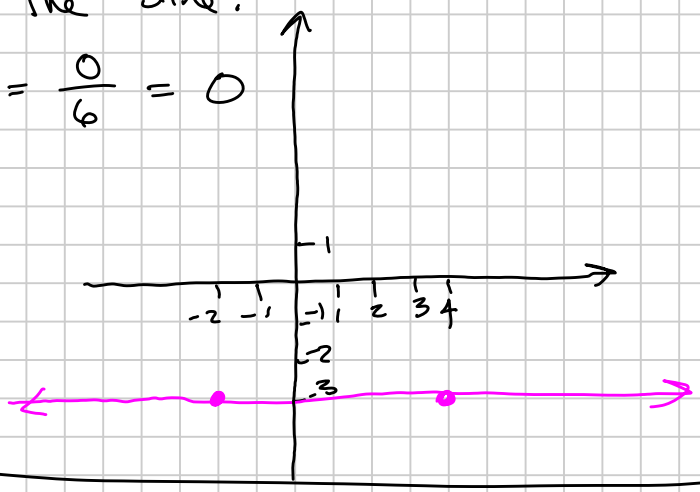
Ex: Find the slope of the line containing $(-2,-3)$ and $(4,-3)$.

Write the eqn of the line.

$$\text{Slope: } m = \frac{-3-(-3)}{4-(-2)} = \frac{-3+3}{4+2} = \frac{0}{6} = 0$$

The slope is 0.

The equation of this line is $y = -3$.



Important

The slope of a vertical line is undefined

The slope of a horizontal line is 0.

Important:

The equation of a vertical line has the form $x = k$, k a constant.

The equation of a horizontal line has the form $y = k$, k a constant.

Could we use $y = mx + b$ with vertical and horizontal lines? Yes for horizontal, no for vertical.

Horizontal: $m = 0$

$y = 0x + b$

$y = 0 + b$

$\boxed{y = b}$

Vertical: Slope is undefined, so there is no m to plug into $y = mx + b$.

Finding the equation of a line given 2 points:

Example: Find the equation of the line containing the points $(-2, 4)$ and $(2, -8)$.

Method 1: using slope-intercept form

In order to use $y = mx + b$, we need the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8 - 4}{2 - (-2)} = \frac{-12}{2 + 2} = \frac{-12}{4} = -3$$

$y = mx + b$ Still need to find b.

Plug in the m, and either of the known points:

$m = -3 \Rightarrow y = -3x + b$

Using $(-2, 4)$: $x = -2$, $y = 4 \Rightarrow$ $4 = -3(-2) + b$

Solve for b: $4 = 6 + b$

$-2 = b$

Eqn of line: $\boxed{y = -3x - 2}$

Another approach: Use the point-slope form of a line.

Point-slope form of a line:

$$y - y_1 = m(x - x_1),$$

where m is the slope, and (x_1, y_1) is any point on the line.

Method 2: Use point-slope form.

Same problem: Find eqn of line through $(-2, 4)$ and $(2, -8)$.

Find slope: $m = \frac{-8-4}{2-(-2)} = \frac{-12}{2+2} = \frac{-12}{4} = -3$

Put $m = -3$ and either of the known points into the point-slope form:

$$y - y_1 = m(x - x_1)$$

$m = -3$, $x_1 = -2$, $y_1 = 4$ $\Longrightarrow$ $y - 4 = -3(x - (-2))$

$$y - 4 = -3(x + 2)$$

$$y - 4 = -3x - 6$$

$$\boxed{y = -3x - 2}$$ same as before!

Example:

Find the equation of the line containing the points $(-3, -4)$ and $(5, -8)$

$$m = \frac{-8-(-4)}{5-(-3)} = \frac{-8+4}{5+3} = \frac{-4}{8} = -\frac{1}{2}$$

Using $y = mx + b$:

$$y = -\frac{1}{2}x + b$$ still need b.

$x = 5$, $y = -8$ $\Longrightarrow$ $-8 = -\frac{1}{2}(5) + b$

$$-8 = -\frac{5}{2} + b$$

$$-8 + \frac{5}{2} = b$$

See next page

Previous example cont'd:

$$-\frac{8}{1} + \frac{5}{2} = b$$

$$-\frac{16}{2} + \frac{5}{2} = b$$

$$-\frac{11}{2} = b$$

Write the eqn of the line: $\boxed{y = -\frac{1}{2}x - \frac{11}{2}}$

using Point-slope form: $y - y_1 = m(x - x_1)$

$m = -\frac{1}{2}$, $x = -3$, $y = -4 \Rightarrow$ $y - (-4) = -\frac{1}{2}(x - (-3))$

$$y + 4 = -\frac{1}{2}(x+3)$$

$$y + 4 = -\frac{1}{2}x - \frac{1}{2}(3)$$

$$y + 4 = -\frac{1}{2}x - \frac{3}{2}$$

$$y = -\frac{1}{2}x - \frac{3}{2} - \frac{4}{1}$$

$$y = -\frac{1}{2}x - \frac{3}{2} - \frac{8}{2}$$

$$\boxed{y = -\frac{1}{2}x - \frac{11}{2}}$$

Check answers: $(-3, -4)$ and $(5, -8)$

$x = -3, y = -4 \Rightarrow$

$$-4 = -\frac{1}{2}(-3) - \frac{11}{2}$$

$$-4 = \frac{3}{2} - \frac{11}{2}$$

$$-4 = -\frac{8}{2}$$

$$-4 = -4 \checkmark$$

$x = 5, y = -8 \Rightarrow$

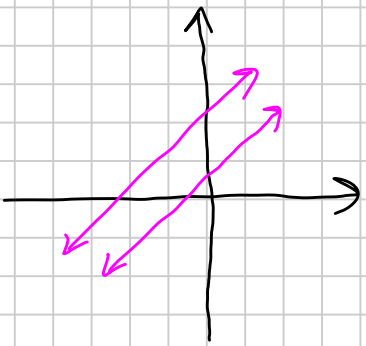
$$-8 = -\frac{1}{2}(5) - \frac{11}{2}$$

$$-8 = -\frac{5}{2} - \frac{11}{2}$$

$$-8 = -\frac{16}{2}$$

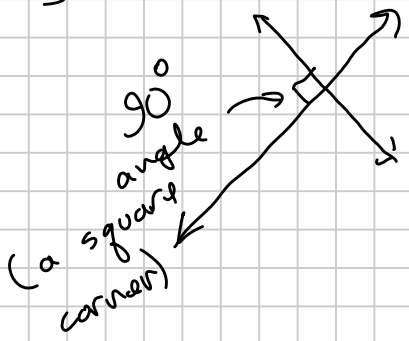
$$-8 = -8 \checkmark$$

Parallel and Perpendicular Lines



* Parallel lines have the same slopes
  (If 2 lines are parallel, their slopes are equal)

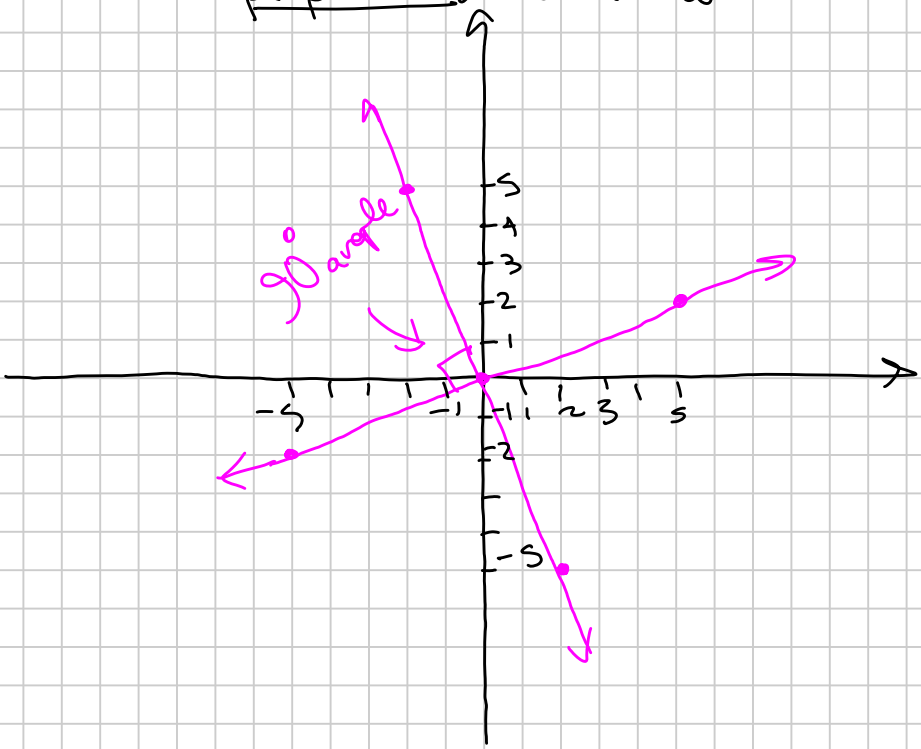
* The slopes of perpendicular lines are opposite reciprocals.



Example: If a line has slope $\frac{2}{5}$, then

1) all lines parallel to it also have slope $\frac{2}{5}$.
2) All lines perpendicular to it will have slope $-\frac{5}{2}$.

Why?



Example: Find the equation of the line that includes the point $(-1, 2)$ and is parallel to the line with equation $4x - 2y = 6$.

Find slope of the given line: $4x - 2y = 6$

Write as $y = mx + b$: $-2y = -4x + 6$

$$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{6}{-2}$$

$$y = 2x - 3$$

Slope of given line: $m = 2 = \frac{2}{1}$

Slope of requested line: $m = 2$ also

(because they are parallel)

Find eqn of the requested line:

Use $m = 2$, $x_1 = -1$, $y_1 = 2$:

Using point-slope form: $y - y_1 = m(x - x_1)$

$$y - 2 = 2(x - (-1))$$

$$y - 2 = 2(x + 1)$$

$$y - 2 = 2x + 2$$

$$\boxed{y = 2x + 4}$$

Example: Find the equation of the line that includes the point $(4,5)$ and is perpendicular to the line $-3x-5y=7$.

Find slope of given line: $-3x-5y=7$

write as $y=mx+b$:

$$-5y = 3x+7$$

$$\frac{-5y}{-5} = \frac{3x}{-5} + \frac{7}{-5}$$

$$y = -\frac{3}{5}x - \frac{7}{5}$$

Slope of given line: $-\frac{3}{5} = m_1$

Slope of requested line: $+\frac{5}{3} = m$

(because they are perpendicular)

Find equation of requested line:

Use $m = \frac{5}{3}$, $x = 4$, $y = 5$:

Using slope-intercept form: $y = mx+b$

$$5 = \frac{5}{3}(4) + b$$

$$5 = \frac{20}{3} + b$$

$$5 - \frac{20}{3} = b$$

$$\frac{15}{3} - \frac{20}{3} = b$$

$$-\frac{5}{3} = b$$

Note: our line will have the form $y = \frac{5}{3}x + b$

The equation of the requested line is $\boxed{y = \frac{5}{3}x - \frac{5}{3}}$