

4.1: Multiplication with Exponents

Note Title

2/25/2016

Properties (Laws) of Exponents

$$1) x^a x^b = x^{a+b}$$

$$\underline{\text{Ex:}} \quad x^2 x^3 = (xx)(xxx) = x^5$$

$$2) (x^a)^b = x^{ab}$$

$$\underline{\text{Ex:}} \quad (x^2)^3 = x^2 x^2 x^2 = (xx)(xx)(xx) = x^6$$

$$3) \frac{x^a}{x^b} = x^{a-b}$$

$$\underline{\text{Ex:}} \quad \frac{x^6}{x^2} = \frac{\cancel{xxx} \cancel{xx} \cancel{x}}{\cancel{xx}} = \frac{xxx}{1} = \frac{x^4}{1} = x^4$$

$$4) (xy)^a = x^a y^a$$

$$\underline{\text{Ex:}} \quad (xy)^3 = (xy)(xy)(xy) = xxx yyy = x^3 y^3$$

$$5) \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

$$\underline{\text{Ex:}} \quad \left(\frac{x}{y}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{xxx}{yyy} = \frac{x^3}{y^3}$$

$$\underline{\text{Example:}} \quad x^3 x^7 = x^{3+7} = \boxed{x^{10}}$$

$$\underline{\text{Ex:}} \quad (x^2)^7 = x^{2(7)} = \boxed{x^{14}}$$

$$\underline{\text{Ex:}} \quad \frac{x^{15}}{x^6} = \frac{x^9}{1} = \boxed{x^9}$$

Powers of Positive and Negative Numbers

Example: $(-5)^2 = (-5)(-5) = \boxed{25}$

$-5^2 = -(5)(5) = \boxed{-25}$

Note: $0 - 5^2 = 0 - 25 = \boxed{-25}$

Know these powers:

$$\begin{array}{r} 2^7 \\ 8 \overline{) 16} \\ 16 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 16 \\ 1 \overline{) 16} \\ 16 \\ \hline 0 \end{array}$$

$$\begin{aligned} 2^2 &= 4 \\ 2^3 &= 8 \\ 2^4 &= 16 \\ 2^5 &= 32 \\ 2^6 &= 64 \end{aligned}$$

$$\begin{aligned} 2^7 &= 128 \\ 2^8 &= 256 \\ 2^9 &= 512 \\ 2^{10} &= 1024 \end{aligned}$$

$$\begin{aligned} 10^2 &= 100 \\ 10^3 &= 1000 \\ 10^4 &= 10\,000 \\ 10^5 &= 100\,000 \\ 10^6 &= 1\,000\,000 \end{aligned}$$

$$\begin{aligned} 3^2 &= 9 \\ 3^3 &= 27 \\ 3^4 &= 81 \\ 3^5 &= 243 \\ 3^6 &= 729 \end{aligned}$$

$$\begin{aligned} 4^2 &= 16 \\ 4^3 &= 64 \\ 4^4 &= 256 \end{aligned}$$

$$\begin{aligned} 8^2 &= 64 \\ 8^3 &= 512 \end{aligned}$$

$$\begin{aligned} 5^2 &= 25 \\ 5^3 &= 125 \\ 5^4 &= 625 \end{aligned}$$

$$\begin{aligned} 6^2 &= 36 \\ 6^3 &= 216 \end{aligned}$$

$$\begin{aligned} 7^2 &= 49 \\ 7^3 &= 343 \end{aligned}$$

$$\begin{aligned} 9^2 &= 81 \\ 9^3 &= 729 \end{aligned}$$

Note: $4^4 = (2^2)^4 = 2^8 = 256$

$$\begin{array}{r} 1^2 \\ 1 \overline{) 1} \\ 1 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 3^6 \\ 3 \overline{) 216} \\ 216 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 6^4 \\ 6 \overline{) 1296} \\ 1296 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 3^8 \\ 3 \overline{) 6561} \\ 6561 \\ \hline 0 \end{array}$$

Powers of negative numbers

Example: $(-2)^6 = \underbrace{(-2)(-2)}_{+} \underbrace{(-2)(-2)}_{+} \underbrace{(-2)(-2)}_{+} = \boxed{64}$

$$(-2)^7 = \underbrace{(-2)(-2)}_{+} \underbrace{(-2)(-2)}_{+} \underbrace{(-2)(-2)}_{+} \underbrace{(-2)}_{-} = \boxed{-128}$$

$$-2^6 = \boxed{-64}$$

$$-2^7 = \boxed{-128}$$

Important:

- * If you raise a negative number to an even exponent, you get a positive number.
- * If you raise a negative number to an odd exponent, you get a negative number.

Ex: $(-3)^4 = \boxed{81}$

$$-5^3 = \boxed{-125}$$

$$(-5)^3 = \boxed{-125}$$

$$-3^4 = \boxed{-81}$$

Ex: $-(-2)^4 = \boxed{-16}$

$$-(-1)^{11} = -(-1) = \boxed{1}$$

Ex: $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \boxed{\frac{27}{64}}$

Note: $1^2 = 1$

$1^3 = 1$

$1^4 = 1$

$1^5 = 1$

Ex: $\left(-\frac{1}{3}\right)^4 = + \frac{1^4}{3^4} = \boxed{\frac{1}{81}}$

Ex: $\frac{3^{13}}{3^{11}} = 3^{13-11} = 3^2 = \boxed{9}$ ← Property #3

$\therefore 2^3 = 1$

Ex: $\frac{2^5}{3^4} = \boxed{\frac{32}{81}}$

Ex: $\frac{4^3}{2^5} = \frac{64}{32} = \boxed{2}$ or $\frac{4^3}{2^5} = \frac{(2^2)^3}{2^5} = \frac{2^6}{2^5} = 2^{6-5} = 2^1 = \boxed{2}$

$\frac{4^8}{2^{12}} = \frac{(2^2)^8}{2^{12}} = \frac{2^{16}}{2^{12}} = 2^{16-12} = 2^4 = \boxed{16}$

Ex: $2^4 + 2^5 = 16 + 32 = \boxed{48}$

Note:

$2^4 + 2^5 \neq 2^9$

Ex: $3x^3x^5 = \boxed{3x^8}$ ← Prop. #1

Ex: $(4y^3)(-3y^7) = -12y^{3+7} = \boxed{-12y^{10}}$ ← Prop #1

Ex: $(x^4y^5)^3 = (x^4)^3(y^5)^3 \leftarrow \text{Prop \#4}$
 $= \boxed{x^{12}y^{15}}$ ← Prop #2

Ex: $\left(\frac{y^1}{z^2}\right)^3 = \frac{(y^1)^3}{(z^2)^3} \rightarrow \text{Prop \#5}$

$= \boxed{\frac{y^{12}}{z^6}} \rightarrow \text{Prop. \#2}$

Ex: $(-2x^2)^5 = (-2)^5 x^{10}$

$= \boxed{-32x^{10}}$

Ex: $(-3xy^3)^4 = (-3)^4 x^4 y^{12} = \boxed{81x^4y^{12}}$

Ex: $(-xy^2z)^7(-5xz^2)^2$

$= (-1)^7 x^7 y^{14} z^7 (-5)^2 x^2 z^4$

$= -1(25) x^9 y^{14} z^{11}$

$= \boxed{-25x^9y^{14}z^{11}}$

Ex: $\frac{(3x^2yz)(-x^1y^2z^3)^4}{(-5xy)^2} = \frac{3x^2yz(-1)^4x^{16}y^8z^{12}}{(-5)^2x^2y^2}$

$= \frac{3(1)x^{18}y^9z^{13}}{25x^2y^2}$

$= \boxed{\frac{3x^{16}y^7z^{13}}{25}} = \boxed{\frac{3}{25}x^{16}y^7z^{13}}$

4.2 Division w/ Exponents and Negative Exponents

Definition of a negative exponent:

$$x^{-n} = \frac{1}{x^n}$$

Ex: $x^4 y^{-3} z^{-2} = \frac{x^4 y^{-3} z^{-2}}{1} =$

$$\boxed{\frac{x^4}{y^3 z^2}}$$

Ex: $5^{-2} = \frac{5^{-2}}{1} = \frac{1}{5^2} =$

$$\boxed{\frac{1}{25}}$$

Ex: $\frac{3x^{-2}y}{5^{-2}y^{-3}z^2w^{-1}} = \frac{3}{x^2} y (5)^2 y^3 \frac{w^1}{z^2}$

$$= \frac{3(25)y^4w}{x^2z^2} = \boxed{\frac{75y^4w}{x^2z^2}}$$