4.2: Division with Exponents and Negative Exponents

Note Title 3/1/2016

For now: skip scientific notation in 4.1 .... we'll do it at the end of 4.2

Definition of a Negative Exponent

$$x^{-n} = \frac{1}{x^n}$$

Note: $x^{-n} = \frac{x^{-n}}{1} = \frac{1}{x^n}$

$x^{-1} = \frac{x^{-1}}{1} = \frac{1}{x^1} = \frac{1}{x}$ (so x^{-1} is the reciprocal of x)

Also: $\frac{1}{x^{-n}} = \frac{1}{\frac{1}{x^n}} = \frac{1}{1} \cdot \frac{x^n}{1} = \frac{x^n}{1}$

Compare these

Changing the sign on the exponent moves the number (either a constant or a variable) to the other part of the fraction.

In other words, changing the sign on the exponent moves it from the numerator to the denominator, or vice versa

In all the following examples, instructions are: Simplify. Write answers so they do not contain negative exponents.

Example: (simplify) $5^{-2} = \frac{5^{-2}}{1} = \frac{1}{5^2} = \boxed{\frac{1}{25}}$

Ex: $-2^{-4} = -\frac{2^{-4}}{1} = -\frac{1}{2^4} = \boxed{-\frac{1}{16}}$

Ex: $(-3)^{-4} = \frac{(-3)^{-4}}{1} = \frac{1}{(-3)^4} = \boxed{\frac{1}{81}}$

Ex: $-(-3)^{-5} = -\frac{(-3)^{-5}}{1} = -\frac{1}{(-3)^5} = -\frac{1}{-243}$

$= \boxed{\frac{1}{243}}$

Note: $-\frac{-1}{-243} = \boxed{-\frac{1}{243}}$

Ex: $\left(\frac{3}{4}\right)^{-2} = \frac{3^{-2}}{4^{-2}} = \frac{4^2}{3^2} = \boxed{\frac{16}{9}}$

OR $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^{2} = \frac{4^2}{3^2} = \boxed{\frac{16}{9}}$

Ex: $\frac{3}{5^{-3}} = \frac{3 \cdot 5^3}{1} = \frac{3 \cdot 125}{1} = \boxed{375}$

Ex: $2x^{-5} = \frac{2x^{-5}}{1} = \boxed{\frac{2}{x^5}}$

Ex: $(2x^{-5}) = 2x^{-5} = \frac{2x^{-5}}{1} = \boxed{\frac{2}{x^5}}$

Same as before

Oops!

Ex: $(2x)^{-5} = \frac{(2x)^{-5}}{1} = \frac{2^{-5}x^{-5}}{1} = \frac{1}{2^5x^5}$

$= \boxed{\frac{1}{32x^5}}$

Ex: $(-5x^3yz^{-2})^{-2} = (-5)^{-2}x^{-6}y^{-2}z^4$

$= \frac{(-5)^{-2}x^{-6}y^{-2}z^4}{1}$

$= \frac{z^4}{(-5)^2x^6y^2}$

$= \boxed{\frac{z^4}{25x^6y^2}}$

Ex: $\left(\frac{2x^2}{y^4}\right)^{-3} = \frac{2^{-3}x^{-6}}{y^{-12}} = \frac{y^{12}}{2^3x^6} = \boxed{\frac{y^{12}}{8x^6}}$

Example:

$$\frac{-3x^{-5}\left(-x^{-1}y^{3}z^{-2}\right)^{-4}}{(3x^{2}y^{4})^{-2}(2xy^{-3})^{6}}$$

$$= \frac{-3x^{-5}(-1)^{-4}x^{4}y^{-12}z^{8}}{(3)^{-2}x^{-4}y^{-8}\cdot(2)^{6}x^{6}y^{-18}}$$

$$= \frac{-3}{x^{5}(-1)^{4}}\cdot\frac{x^{4}}{y^{12}}\cdot\frac{z^{8}(3)^{2}x^{4}y^{8}}{(2)^{6}x^{6}}\cdot\frac{y^{18}}{1}$$

$$= \frac{-3(9)x^{4}z^{0}x^{4}y^{8}y^{18}}{(1)(64)x^{5}y^{12}x^{6}}$$

$$= \frac{-27x^{8}y^{26}z^{8}}{64x^{11}y^{12}}$$

$$= \boxed{-\frac{27y^{14}z^{8}}{64x^{3}}}$$

Important Fact:

$x^0 = 1$, as long as $x \neq 0$. ✱

So: $2^0 = 1$

$(-5)^0 = 1$

$(xy)^0 = 1$ as long as $x \neq 0, y \neq 0$

Note: 0^0 is undefined.

→ Here's why $x^0 = 1$: Start with $x = 2$. What is 2^0?

$$\frac{2^3}{2^3} = \frac{8}{8} = 1.$$

From the properties of exponents (Prop. #3),

$$\frac{2^3}{2^3} = 2^{3-3} = 2^0$$

must be equal

So, 2^0 must equal 1.

Prop #3 of Exponents:

$$\frac{x^a}{x^b} = x^{a-b}$$

Same approach for other bases:

$5^0 = 1$

$(-4)^0 = 1$

$(4x^2y^{-8}z^4)^0 = 1$

as long as $x \neq 0, y \neq 0, z \neq 0$

Scientific Notation: We write numbers so they have exactly 1 digit to the left of the decimal point, and are multiplied by a power of 10.

Ex: write 3254 in scientific notation.

$3254 = \boxed{3.254 \times 10^3}$

(to go from 3254 to 3.254 requires moving decimal 3 slots

Ex: write 52,683 in scientific notation.

$52{,}683 = \boxed{5.2683 \times 10^4}$

Ex: write 0.0002794 in scientific notation,

$0.0002794 = \boxed{2.794 \times 10^{-4}}$

0.0002794

4 slots

Ex: write the number without scientific notation.

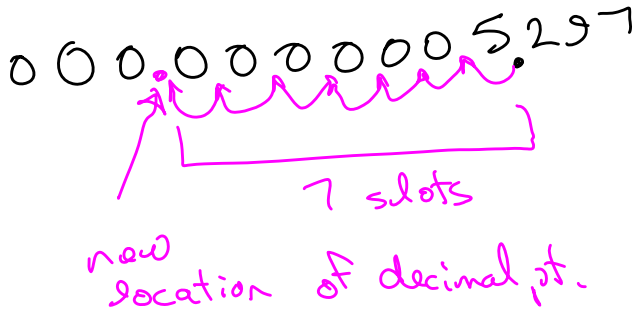
$2.7135 \times 10^3 = \boxed{2713.5}$

2.7135 ⟷ 2713.5

3 slots

Ex: Write 5.297×10^{-7} without scientific notation.

$5.297 \times 10^{-7} = \boxed{0.0000005297}$



Ex: Write 6.3×10^{9} without scientific notation.

scratchwork

$6.3 \times 10^{9} = \boxed{6,300,000,000}$