4.3: Operations with Monomials

Note Title 3/3/2016

Polynomial (Polynomial Expression): Expression composed of terms added together, in which each term has variables to positive integers only.

Examples of Polynomials: $2x^2 - 8x + 12$

$27x - 3$

$3xy^2 - 4yx^3 + 6z$

$\frac{2}{5}x^3 - \frac{1}{4}x^2 + \frac{4}{3}x - \frac{9}{10}$

Not polynomials: $2x^{-4} - 5x^{-3}$

$\frac{4}{x^5} + \frac{1}{x^4} - \frac{6}{x}$ (Not a polynomial... could write as $4x^{-5} + x^{-4} - 6x^{-1}$)

$\sqrt{x}$ (could write as $x^{1/2}$)

☆☆ **Degree of a polynomial in 1 variable:** the degree is the largest exponent.

Example: $3x^5 - 2x^6 + 12x^2 - 22$

$\boxed{\text{The degree is 6.}}$

Ex: $2x - 5x^7$

$\boxed{\text{The degree is 7.}}$ (It is a 7^{th} degree polynomial)

Degree of a polynomial with 2 or more variables:

Largest total exponent in a term.

Ex: What is the degree?

$$\underbrace{2x^2y^3}_{\substack{\text{total exponent}\\ =2+3=5}} + \underbrace{5x^3y^4}_{\substack{\text{total}\\ \text{exponent}\\ =3+4=7}} - \underbrace{7xy^9}_{\substack{\text{total}\\ \text{exponent}\\ =1+9=10}} - \underbrace{8y^4}_{\text{total exponent } =4}$$

Largest

$\boxed{\text{The degree is } 10}$

Special names for certain polynomials

Polynomial with 3 terms: Trinomial

Polynomial with 2 terms: Binomial

Polynomial with 1 term: Monomial

Example: $7 - x^4$ is a 4^{th} degree binomial.

$5x^2 + 9x - 2$ is a 2^{nd} degree trinomial.

$3x^4y^6$ is a 10^{th} degree monomial.

Multiplying Monomials:

Ex: $(-4x^3)(5y^2) = -20x^3y^2$

Ex: $8x^3(-2x^4) = \boxed{-16x^7}$

Ex: $(2x^3y)(5x^4y^3z)$

$= \boxed{10x^7y^4z}$

Ex: $\dfrac{\cancelto{8}{24}x^6}{-\cancelto{1}{3}x^2} = -\dfrac{8x^4}{1} = \boxed{-8x^4}$

Ex: $\dfrac{2x^3}{8x^7} = \boxed{\dfrac{1}{4x^4}}$ could write as $\frac{1}{4}x^{-4}$

In this class, write answers with positive exponents only

may write it this way in future classes (calculus for example)

4.4: Addition and Subtraction of Polynomials

Example: $(3x^2-7x+5)+(-4x^2+8x+1)$ simplify.

$= \underline{3x^2} - \underline{\underline{7x}} + 5 - \underline{4x^2} + \underline{\underline{8x}} + 1$

$= \boxed{-x^2+x+6}$

Ex: Simplify.

$$2x^3 - 5x^2 - 3x + 6 - (-x^2 + 5x - 8)$$

Can Rewrite: $2x^3 - 5x^2 - 3x + 6 - 1(-x^2 + 5x - 8)$

$$= 2x^3 - 5x^2 - 3x + 6 + x^2 - 5x + 8$$

$$= \boxed{2x^3 - 4x^2 - 8x + 14}$$

4.5: Multiplication of Polynomials

Example: $3x^2(5x^2 - 6x - 2)$

$$= 3x^2(5x^2) + 3x^2(-6x) + 3x^2(-2)$$

$$= \boxed{15x^4 - 18x^3 - 6x^2}$$

Ex: $-2x^4(5x^3 - 6x^2 + 12x - 1)$

$$= \boxed{-10x^7 + 12x^6 - 24x^5 + 2x^4}$$

Ex: $-5x^2y^3z(-xy^4z^2 + 8xy - 3xy^5z^3 - 2z^3)$

$$= -5x^2y^3z(-xy^4z^2) - 5x^2y^3z(8xy) - 5x^2y^3z(-3xy^5z^3) - 5x^2y^3z(-2z^3)$$

$$= \boxed{5x^3y^7z^3 - 40x^3y^4z + 15x^3y^8z^4 + 10x^2y^3z^4}$$

Multiplying 2 polynomials that have 2 or more terms

Recall: Distributive property:

$c(a+b) = ca+cb$
$(a+b)(c) = ac+bc$

both equal to $ac+bc$ ($ac=ca$ etc)

Example: $(\underbrace{2x}_{a}\underbrace{-7}_{b})\underbrace{(x+5)}_{c}$

$= \underbrace{2x}_{a}\underbrace{(x+5)}_{c} \underbrace{-7}_{b}\underbrace{(x+5)}_{c}$

$= 2x^2+10x-7x-35$

$= \boxed{2x^2+3x-35}$

OR $\underbrace{(2x-7)}_{c}(x+5)$

$(2x-7)(x)+(2x-7)(5)$

$2x^2-7x+10x-35$

$= \boxed{2x^2+3x-35}$

Ex: $(5x+3)(4x+9)$

$= 5x(4x+9)+3(4x+9)$

$= 20x^2+45x+12x+27$

$= \boxed{20x^2+57x+27}$

Ex: $(2x-1)(6x-5)$

$= 2x(6x-5)-1(6x-5)$

$= 12x^2-10x-6x+5$

$= \boxed{12x^2-16x+5}$

Ex: $(-2x+8)(x^2+7x-4)$

$= -2x(x^2+7x-4) + 8(x^2+7x-4)$

$= -2x^3 - 14x^2 + 8x + 8x^2 + 56x - 32$

$= \boxed{-2x^3 - 6x^2 + 64x - 32}$

Ex: $(x^2-7x-1)(2x^2-5x-6)$

$= x^2(2x^2-5x-6) - 7x(2x^2-5x-6) - 1(2x^2-5x-6)$

$= 2x^4 - 5x^3 - 6x^2$
$\quad -14x^3 + 35x^2 + 42x$
$\quad -2x^2 + 5x + 6$

$= \boxed{2x^4 - 19x^3 + 27x^2 + 47x + 6}$

4.6: Special Products

The "FOIL" Method (for multiplying 2 binomials)

F: First

O: Outer

I: Inner

L: Last

F = first L = last

Example: $(2x-5)(3x+4)$

I = inner

O = outer

$= 6x^2 + 8x - 15x - 20$

$= \boxed{6x^2 - 7x - 20}$

Compare to before: $(2x-5)(3x+4) = 2x(3x+4) - 5(3x+4)$
$= 6x^2 + 8x - 15x - 20$
$= 6x^2 - 7x - 20$

Ex: $(-6x+3)(4y-7)$

$\boxed{-24xy+42x+12y-21}$ (No like terms)

Perfect Squares:

Ex: $(x-8)^2$

$= (x-8)(x-8)$

$= x^2-8x-8x+64$

$= \boxed{x^2-16x+64}$

Caution:

$(x-8)^2 \neq x^2-64$

$(x-8)^2 \neq x^2+64$

Note:

$(8x)^2 = 64x^2$

Perfect Square Pattern (no need to memorize)

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2-2ab+b^2$

Ex: $(2x+5)^2$

$= 4x^2+20x+25$

$(2x+5)(2x+5)$

$= 4x^2+10x+10x+25$

$= 4x^2+20x+25$

Ex: $(x-7)(x+7)$

$= x^2+7x-7x-49$

$= \boxed{x^2-49}$

Difference of 2 squares pattern

$$\boxed{(a+b)(a-b) = a^2 - b^2}$$

$$(a+b)(a-b) = a^2 - \cancel{ab} + \cancel{ab} - b^2$$
$$= a^2 - b^2$$