Review Problems from 4.3 - 4.6

Note Title 3/8/2016

Example: multiply

$(-3x-4)(-x+8)$

Ex 2 multiply

$4x(x-5)(x+6)$

Ex3: $(3x-2y+7)(5x-7y-2)$

Ex 6: multiply.

$(2x-3)^3$

Ex4: $(3x^2-5x)(4x+3)$

Ex 5: $(2x-5)(2x+5)$

Ex3: $(3x-2y+7)(5x-7y-2)$

$= 3x(5x-7y-2)-2y(5x-7y-2)+7(5x-7y-2)$

$= 15x^2-21xy-6x-10xy+14y^2+4y+35x-49y-14$

$= \boxed{15x^2-31xy+29x+14y^2-45y-14}$

Ex4: $(3x^2-5x)(4x+3)$

$= 3x^2(4x+3) - 5x(4x+3)$

$= 12x^3 + 9x^2 - 20x^2 - 15x$

$= \boxed{12x^3 - 11x^2 - 15x}$

Ex6: $(2x-3)^3$

$= (2x-3)(2x-3)(2x-3)$

$= (4x^2 - 6x - 6x + 9)(2x-3)$

$= (4x^2 - 12x + 9)(2x-3)$

$= (2x-3)(4x^2 - 12x + 9)$

$= 2x(4x^2 - 12x + 9) - 3(4x^2 - 12x + 9)$

$= 8x^3 - 24x^2 + 18x$

$\quad - 12x^2 + 36x - 27$

$= 8x^3 - 36x^2 + 54x - 27$

4.7: Dividing a polynomial by a monomial

Example: Divide.

$$\frac{6x^4 - 8x^3 + 12x^2 - 9}{2x^2}$$

Note:

$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7}$$

$$= \frac{5}{7}$$

Split into separate fractions:

$$\frac{\cancel{6}^{3}x^4}{\cancel{2}_{1}x^2} - \frac{\cancel{8}^{4}x^3}{\cancel{2}_{1}x^2} + \frac{\cancel{12}^{6}\cancel{x^2}^{1}}{\cancel{2}_{1}\cancel{x^2}_{1}} - \frac{9}{2x^2}$$

$$= \frac{3x^2}{1} - \frac{4x}{1} + \frac{6}{1} - \frac{9}{2x^2}$$

$$= \boxed{3x^2 - 4x + 6 - \frac{9}{2x^2}}$$

Example:

$$\frac{2x^4y^3 + 8x^3y^3 - 4y^2}{4x^2y}$$

$$= \frac{\cancel{2}^{1}x^4y^3}{\cancel{4}_{2}x^2y} + \frac{\cancel{8}^{2}x^3y^3}{\cancel{4}_{1}x^2y} - \frac{\cancel{4}^{1}y^2}{\cancel{4}_{1}x^2y}$$

$$= \frac{1x^2y^2}{2} + \frac{2xy^2}{1} - \frac{y}{x^2}$$

$$= \boxed{\frac{x^2y^2}{2} + 2xy^2 - \frac{y}{x^2}} \quad \text{OR} \quad \boxed{\frac{1}{2}x^2y^2 + 2xy^2 - \frac{y}{x^2}}$$

4.8: Dividing a Polynomial by a Polynomial

(where divisor has 2 or more terms)

Polynomial Long Division

Examples: Divide $\frac{379}{12}$

$$\begin{array}{r} 31 \\ 12\,\overline{)\,379} \\ -\,36 \\ \hline 19 \\ -\,12 \\ \hline 7 \end{array}$$

So, $\frac{379}{12} = \boxed{31 + \frac{7}{12}}$ (we usually write this as $31\frac{7}{12}$)

Terminology Review

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor}\,\overline{)\,\text{Dividend}} \\ \vdots \\ \hline \text{Remainder} \end{array}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$

(multiplying both sides by Divisor)

Example: Divide.

$$\frac{3x^2+7x+9}{x+2}$$

Note: If we write as separate fractions, we can't simplify any of them:

$$\frac{3x^2}{x+2}+\frac{7x}{x+2}+\frac{9}{x+2}$$

$$\frac{3x^2}{x} \qquad \frac{x}{x}$$

$$\begin{array}{r|l}
 & 3x + 1 \\
\hline
x+2 & 3x^2 + 7x + 9 \\
 & -(3x^2 + 6x) \\
\hline
 & 0 + \quad x + 9 \\
 & -(x + 2) \\
\hline
 & 0 + \quad 7
\end{array}$$

$x(?)=3x^2$
Answer: $3x$

$3x(x+2)=3x^2+6x$

$x(?)=x$
Answer: 1

$1(x+2)=x+2$

$$\frac{3x^2+7x+9}{x+2} = \boxed{3x+1+\frac{7}{x+2}}$$

answer

Check: (Quotient)(Divisor) + Remainder $\overset{?}{=}$ Dividend

$$(3x+1)(x+2)+7$$
$$= 3x^2+6x+1x+2+7$$
$$= 3x^2+7x+2+7$$
$$= 3x^2+7x+9 \checkmark$$

Note: If you substitute $x=10$ throughout this problem, you get

$$\frac{379}{12} = 31+\frac{7}{12}$$

Example: Divide

$$\frac{x^3-6x^2+x+14}{x-5}$$

$$\frac{x^3}{x} \qquad \frac{-x^2}{x} \qquad \frac{-4x}{x}$$

$$
\begin{array}{r|rrrr}
 & x^2 & -x & -4 & \\
\hline
x-5 & x^3 & -6x^2 & +x & +14 \\
 & -(x^3 & -5x^2) & & \\
\hline
 & & -x^2 & +x & \\
 & & -(-x^2 & +5x) & \\
\hline
 & & & -4x & +14 \\
 & & & -(-4x & +20) \\
\hline
 & & & & -6
\end{array}
$$

$\leftarrow x^2(x-5)$

$\leftarrow -x(x-5)$

$\leftarrow -4(x-5)$

$$\frac{x^3-6x^2+x+14}{x-5} = \boxed{x^2-x-4+\frac{-6}{x-5}}$$

usually written as:

$$\boxed{x^2-x-4-\frac{6}{x-5}}$$

Check: (Quotient)(Divisor) + Remainder $\stackrel{?}{=}$ Dividend

$$(x^2-x-4)(x-5)-6$$

$$= (x-5)(x^2-x-4)-6$$

$$= x^3-x^2-4x-5x^2+5x+20-6 = x^3-6x^2+x+14$$

✓ OK

Important

We put in 0's as "place holders" for missing powers of x

$$\frac{x^4 - 6x^2 + x}{x-3}$$

$$x-3 \overline{\big)\, x^4 + 0x^3 - 6x^2 + x + 0}$$