

4.8: Polynomial Long Division (Continued)

Note Title

3/22/2016

Example: Divide

$$\frac{8x^3 - 10x^2 + x - 2}{2x - 1}$$

$$\begin{array}{r}
 \begin{array}{r}
 \frac{8x^3}{2x} \quad \frac{-6x^2}{2x} \quad \frac{-2x}{2x} \\
 \downarrow \quad \downarrow \quad \downarrow \\
 4x^2 - 3x - 1
 \end{array} \\
 2x-1 \overline{) 8x^3 - 10x^2 + x - 2} \\
 \underline{+ \quad - \quad + \quad} \\
 \underline{8x^3 - 4x^2} \\
 -6x^2 + x \\
 \underline{- \quad + \quad - \quad} \\
 \underline{-6x^2 + 3x} \\
 -2x - 2 \\
 \underline{+ \quad - \quad + \quad -} \\
 \underline{-2x + 1} \\
 -3
 \end{array}$$

$$\leftarrow 4x^2(2x-1) = 8x^3 - 4x^2$$

$$\leftarrow -3x(2x-1) = -3x^2 + 3x$$

$$\leftarrow -1(2x-1) = -2x + 1$$

Write answer:

$$\frac{8x^3 - 10x^2 + x - 2}{2x - 1}$$

$$= 4x^2 - 3x - 1 + \frac{-3}{2x - 1}$$

$$= 4x^2 - 3x - 1 - \frac{3}{2x - 1}$$

both are correct

Check your answer:

$$(2x-1)(4x^2-3x-1) - 3$$

$$= 8x^3 - 6x^2 - 2x - 4x^2 + 3x + 1 - 3$$

$$= 8x^3 - 10x^2 + x - 2 \quad \checkmark \text{ OK}$$

Example: Divide $\frac{3x^4 - 50x^2 - x}{x - 4}$

$$\begin{array}{r}
 3x^3 + 12x^2 - 2x - 9 \\
 x-4 \overline{) 3x^4 + 0x^3 - 50x^2 - x + 0} \\
 \underline{+ \quad - \quad +} \quad (3x^4 - 12x^3) \\
 12x^3 - 50x^2 \\
 \underline{+ \quad - \quad +} \quad (12x^3 - 48x^2) \\
 -2x^2 - x \\
 \underline{+ \quad + \quad -} \quad (-2x^2 + 8x) \\
 -9x + 0 \\
 \underline{+ \quad + \quad -} \quad (-9x + 36) \\
 -36
 \end{array}$$

Check: $(x-4)(3x^3+12x^2-2x-9)-36$

$$\begin{aligned}
 &= 3x^4 + 12x^3 - 2x^2 - 9x \\
 &\quad - 12x^3 - 48x^2 + 8x + 36
 \end{aligned}$$

$$= 3x^4 - 50x^2 - x \quad \checkmark \text{ ok}$$

$$\frac{3x^4 - 50x^2 - x}{x - 4} = \boxed{3x^3 + 12x^2 - 2x - 9 + \frac{-36}{x-4}}$$

OR

$$\boxed{3x^3 + 12x^2 - 2x - 9 - \frac{36}{x-4}}$$

Ex:

Divide.

$$\frac{5x^3 + 21x^2 - 16}{5x - 4}$$

$$\begin{array}{r} x^2 + 5x + 4 \\ 5x - 4 \overline{) 5x^3 + 21x^2 + 0x - 16} \\ \underline{+ \quad - \quad +} \quad (5x^3 - 4x^2) \end{array}$$

$$\begin{array}{r} 25x^2 + 0x \\ \underline{+ \quad - \quad +} \quad (25x^2 - 20x) \end{array}$$

$$\begin{array}{r} 20x - 16 \\ \underline{+ \quad - \quad +} \quad (20x - 16) \\ 0 \end{array}$$

Check: $(5x - 4)(x^2 + 5x + 4) + 0$

$$= 5x^3 + 25x^2 + 20x - 4x^2 - 20x - 16 + 0$$

$$= 5x^3 + 21x^2 - 16 \quad \checkmark$$

$$\frac{5x^3 + 21x^2 - 16}{5x - 4} = x^2 + 5x + 4 + \frac{0}{5x - 4}$$
$$= \boxed{x^2 + 5x + 4}$$

Ex.:

$$\frac{10x^3 + 21x^2 - 5}{2x^2 + 5x + 2}$$

$$\begin{array}{r} 5x - 2 \\ 2x^2 + 5x + 2 \overline{) 10x^3 + 21x^2 + 0x - 5} \\ \underline{+ (-10x^3 + 25x^2 + 10x)} \\ -4x^2 - 10x - 5 \\ \underline{+ (-4x^2 + 10x + 4)} \\ -1 \end{array}$$

Check: $(5x - 2)(2x^2 + 5x + 2) - 1$

$$\begin{aligned} &= 10x^3 + 25x^2 + 10x \\ &\quad - 4x^2 - 10x - 4 - 1 \\ &= 10x^3 + 21x^2 - 5 \quad \checkmark \end{aligned}$$

Write answer:

$$\frac{10x^3 + 21x^2 - 5}{2x^2 + 5x + 2}$$

$$= \boxed{5x - 2 + \frac{-1}{2x^2 + 5x + 2}}$$

$$= \boxed{5x - 2 - \frac{1}{2x^2 + 5x + 2}}$$

Ex:

$$\frac{x^4 - 5x^3 + 6x - 8}{x^2 + 2}$$

$$\begin{array}{r} x^2 - 5x - 2 \\ x^2 + 0x + 2 \overline{) x^4 - 5x^3 + 0x^2 + 6x - 8} \\ \underline{+ \quad - \quad + \quad -} \quad x^4 + 0x^3 + 2x^2 \\ -5x^3 - 2x^2 + 6x \\ \underline{+ \quad + \quad - \quad +} \quad -5x^3 + 0x^2 - 10x \\ -2x^2 + 16x - 8 \\ \underline{+ \quad + \quad - \quad +} \quad -2x^2 + 0x - 4 \\ 16x - 4 \end{array}$$

$$\frac{x^4 - 5x^3 + 6x - 8}{x^2 + 2} = \boxed{x^2 - 5x - 2 + \frac{16x - 4}{x^2 + 2}}$$

Fact: ~~The remainder will generally be a degree that is 1 less~~

The degree of the remainder is generally 1 less than the degree of the divisor

Check:

$$\begin{aligned} & (x^2 + 2)(x^2 - 5x - 2) + 16x - 4 \\ &= x^4 - 5x^3 - 2x^2 \\ & \quad + 2x^2 - 10x - 4 + 16x - 4 \\ &= x^4 - 5x^3 + 6x - 8 \end{aligned}$$