Chapter 5: Factoring Polynomials

Note Title 3/24/2016

$(x+3)(x+4)$ ← factored form

$= x^2+7x+12$ ← unfactored form

In chapter 5, we will be starting with polynomials in unfactored form, and then writing them in factored form.

5.1: Greatest Common Factor and Factoring by Grouping

To find the Greatest Common Factor (GCF):

* Find the largest number that divides into all the coefficients
* Find highest power of each variable that is a factor of all the terms (if a variable shows up in all the terms, choose the smallest power)
* multiply the numerical part and the variable part to get the GCF.

Example: Factor out the GCF.

$15x^4 - 6x^2$

GCF: $3x^2$

$= 3x^2\left(\frac{15x^4}{3x^2} - \frac{6x^2}{3x^2}\right)$

$= \boxed{3x^2(5x^2-2)}$

Check by multiplying:

$3x^2(5x^2-2) = 15x^4 - 6x^2$

✓ OK

Example: Factor.

$48x^3 - 32x$

$= 8x(6x^2 - 4)$

$= 8x(2)(3x^2 - 2)$

$= \boxed{16x(3x^2 - 2)}$

~~G~~CF: $8x$

CF: It's a common factor, but not the greatest common factor

Check it: $8x(6x^2 - 4)$
$= 48x^3 - 32x$ ✓

Check: $16x(3x^2 - 2)$
$= 48x^3 - 32x$ ✓

Ex: Factor

$96x^5 + 80x^4 - 24x^3$

GCF: $8x^3$

24
$4 \cdot 6$
$2 \cdot 2 \cdot 2 \cdot 3$
$= 2^3 \cdot 3$

80
$10 \cdot 8$
$2 \cdot 5 \cdot 2 \cdot 4$
$2 \cdot 5 \cdot 2 \cdot 2 \cdot 2$
$= 2^4 \cdot 5$

96
$32 \cdot 3$
$8 \cdot 4 \cdot 3$
$4 \cdot 2 \cdot 2 \cdot 2 \cdot 3$
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$
$= 2^5 \cdot 3$

So GCF of 24, 80, 96 is $2^3 = 8$

GCF of our polynomial is $8x^3$

$96x^5 + 80x^4 - 24x^3$

$= \boxed{8x^3(12x^2 + 10x - 3)}$

Ex: Factor.

GCF: 6

$24x^3 + 18x^2 - 6$

$= \boxed{6(4x^3 + 3x^2 - 1)}$

Ex: $-30x^4y^3z^2 + 12x^3y^6 - 42x^4y^2z$

GCF: $6x^3y^2$

$= \boxed{6x^3y^2(-5xyz^2 + 2y^4 - 7xz)}$

Check: $6x^3y^2(-5xyz^2 + 2y^4 - 7xz)$

$= -30x^4y^3z^2 + 12x^3y^6 - 42x^4y^2z$

Factoring by Grouping

Example: Factor.

$3x^3 - 7x^2 + 15x - 35$

$= (3x^3 - 7x^2) + (15x - 35)$

$= x^2(3x - 7) + 5(3x - 7)$

$= \boxed{(3x-7)(x^2+5)}$

Check: $(3x-7)(x^2+5)$

$= 3x^3 + 15x - 7x^2 - 35$ ✓

The GCF is 1.

★ It is not correct to say "there is no GCF."

Note:

$x^2y + 5y$

$y(x^2 + 5)$

Ex: $4x^3 - 3x^2 - 36x + 27$

$= (4x^3 - 3x^2) + (-36x + 27)$

$= x^2(4x-3) - 9(4x-3)$

$= (4x-3)(x^2-9)$

$= \boxed{(4x-3)(x+3)(x-3)}$ ← In a later section, we'll learn how to factor x^2-9

Note:
$(x+3)(x-3)$
$= x^2 - 3x + 3x - 9$
$= x^2 - 9$

Difference of 2 squares:
$(a+b)(a-b)$
$= a^2 - b^2$

Ex: $2a^2b - 16a^2 + b - 8$ Factor.

$= (2a^2b - 16a^2) + (b-8)$

$= 2a^2(b-8) + 1(b-8)$

$= \boxed{(b-8)(2a^2+1)}$ Check it!

Also correct:
$\boxed{(2a^2+1)(b-8)}$

Ex: $-16x^2y - 4x^2 + 24yx + 6x$ GCF: $2x$

$= 2x(-8xy - 2x + 12y + 3)$

$= 2x\left[(-8xy - 2x) + (12y + 3)\right]$

$= 2x\left[-2x(\underline{4y+1}) + 3(\underline{4y+1})\right]$

$= 2x\left[(4y+1)(-2x+3)\right]$

$= \boxed{2x(4y+1)(-2x+3)}$

Alternate approach ~~approach~~

$-16x^2y - 4x^2 + 24xy + 6x$

$= (-16x^2y - 4x^2) + (24xy + 6x)$

$= -4x^2(4y+1) + 6x(4y+1)$

$= (4y+1)(\underbrace{-4x^2+6x})$

can factor out $2x$ from this factor

$= (4y+1)(2x)(-2x+3)$

$= \boxed{2x(4y+1)(-2x+3)}$ OR $\boxed{2x(-2x+3)(4y+1)}$

5.2: Factoring Trinomials (when the leading coefficient is 1)

Leading term: (of a polynomial): Term with highest power

Leading coefficient: Coefficient of the leading term

Ex: $2x^2 - 7x^3 + 12x + 5$

Leading term: $-7x^3$

Leading coefficient: -7

Degree: 3

For factoring, we want to write all polynomials in order of descending powers (so leading term is 1st)

Example: Factor

(+) same signs

$x^2 + 6x + 8$

$$\boxed{(x+2)(x+4)}$$

8

1·8

2·4

Check: $(x+2)(x+4)$

$= x^2 + 4x + 2x + 8$

$= x^2 + 6x + 8$

Ex:

$x^2 - 10x + 24$

(+) signs are the same

$$\boxed{(x-4)(x-6)}$$

Check: $x^2 - 6x - 4x + 24$

$= x^2 - 10x + 24$ ✓

signs are the same, so I want a sum of 10

24

1·24
2·12
3·8
4·6

Ex: $x^2 - 10x - 24$

(−) signs are opposite

$$\boxed{(x-12)(x+2)}$$

Check: $x^2 + 2x - 12x - 24$

$= x^2 - 10x - 24$

signs are opposite, so I want a difference of 10

24

1·24
2·12
3·8
4·6

Ex: $x^2 + 10x + 24$

(+) same signs

$(x+4)(x+6)$

$x^2 + 6x + 4x + 24$

$= x^2 + 10x + 24$

24

1·24
2·12
3·8
4·6

Ex: $x^2 + 10x - 24$

(−) signs are opposite

$$\boxed{(x+12)(x-2)}$$

Check: $x^2 - 2x + 12x - 24$

$= x^2 + 10x - 24$

24

1·24
2·12
3·8
4·6