5.2: Factoring Trinomials (when the leading coefficient is 1)

Note Title 3/29/2016

Ex: Factor $x^2 + 16x - 36$

(−) signs are opposite

$$(x - 2)(x + 18)$$

Because my signs are opposite, I want a difference of 16 (middle term must be 16x)

36
1 · 36
2 · 18
3 · 12
4 · 9
6 · 6

Check: $(x-2)(x+18)$
$= x^2 + 18x - 2x - 36$
$= x^2 + 16x - 36$ ✓

Ex: $x^2 - 14x + 48$

(+) signs are the same

$$(x - 6)(x - 8)$$

Because my signs are the same, I want a sum of 14.

48
1 · 48
2 · 24
3 · 16
4 · 12
6 · 8

Check: $x^2 - 8x - 6x + 48$
$= x^2 - 14x + 48$ ✓

Try on your own:

$x^2 + 3x - 54$

$x^2 + 13x + 30$

$x^2 + 7x - 12$

$x^2 - 13x - 30$

$y^2 + 11y - 12$

$x^2 - 13xy + 40y^2$

Example: $x^2 + 3x - 54$ (−) signs are opposite, want difference of 3

$(x+9)(x-6)$

Check: $x^2 - 6x + 9x - 54$

$= x^2 + 3x - 54$ ✓

54
1 · 54
2 · 27
3 · 18
6 · 9

Ex: $x^2 + 13x + 30$ (+) same signs, want sum of 13

$(x+3)(x+10)$

Check

$x^2 + 10x + 3x + 30$

$= x^2 + 13x + 30$ ✓

30
1 · 30
2 · 15
3 · 10
5 · 6

Ex: $x^2 - 13x - 30$ (−) opposite signs, want difference of 13

$(x-15)(x+2)$

check: $x^2 + 2x - 15x - 30$

$= x^2 - 13x - 30$ ✓

Also correct:

$(x+2)(x-15)$

30
1 · 30
2 · 15
3 · 10
5 · 6

Ex: $y^2 + 11y - 12$ (−) signs are opposite, want a difference of 11

$(y-1)(y+12)$

check it!

12
1 · 12
2 · 6
3 · 4

Ex: $x^2+7x-12$ (−) signs are opposite want a difference of 7

12
∧
1 · 12
2 · 6
3 · 4

None of these give a difference of 7.

This polynomial cannot be factored.

This polynomial is prime.

Prime number: a natural number larger than 1 that can only be factored as a product of 1 and itself.

Ex: 7 is prime.

$7 = 1 \cdot 7$, there are no other factorizations (using natural numbers)

Ex: 6 is composite (not prime).

$6 = 1 \cdot 6$

also $6 = 2 \cdot 3$

Prime polynomial: cannot be factored (using integer coefficients) except by factoring out +1 or −1.

Ex: $x^2 - 13xy + 40y^2$ (+) same signs, sum of 13

$$(x - 5y)(x - 8y)$$

40
1·40
2·20
4·10
5·8

Check: $x^2 - 8xy - 5xy + 40y^2$

$= x^2 - 13xy + 40y^2$ ✓

If you have trouble with $x^2 - 13xy + 40y^2$, try factoring $x^2 - 13x + 40$

Ex: $2x^2 + 18x + 28$ Pull out the GCF 1st!

$= 2(x^2 + 9x + 14)$ (+) same signs, sum of 9

$= 2(x + 2)(x + 7)$

14
1·14
2·7

Check: $(x+2)(x+7)$

$= x^2 + 7x + 2x + 14$

$= x^2 + 9x + 14$

Check the whole thing:

$2(x^2 + 9x + 14)$

$= 2x^2 + 18x + 28$ ✓

Ex: $-x^2 + 14x - 40$

$= -1(x^2 - 14x + 40)$ (+) same signs sum of 14

$= -1(x-4)(x-10)$

We want the leading term to be positive before factoring.

So we factor out -1 first

Usually written as

$-(x-4)(x-10)$

OR

$-(x-10)(x-4)$

40

1·40

2·20

4·10

5·8

Check: $(x-4)(x-10)$

$= x^2 - 10x - 4x + 40$

$= x^2 - 14x + 40$ ✓

Ex: $x^2 + 3x - 180$ (−) signs are opposite difference of 3

$(x+15)(x-12)$

$x^2 - 12x + 15x - 180$

$x^2 + 3x - 180$ ✓

1

12

15

60

12

180 ✓

180

1·180

2·90

6·30

9·20

12·15

45·4

60·3

5·36

18·10

180

2·90

2·3·30

2·3·2·15

2·3·2·3·5

5.3: Factoring Trinomials (when the leading coefficient is greater than 1)

Example: $2x^2 + 9x + 10$ ← (+) signs are the same

want sum of $9x$ for middle term

$2x^2$: $2x \cdot x$

10 : $1 \cdot 10$, $2 \cdot 5$

Try $(2x + 1)(x + 10)$

Check: $2x^2 + 20x + 1x + 10$

$= 2x^2 + 21x + 10$ No!

Try $(2x + 10)(x + 1)$

Check: $2x^2 + 2x + 10x + 10$

$= 2x^2 + 12x + 10$ No!

Try $(2x + 2)(x + 5)$

Check: $2x^2 + 10x + 2x + 10$

$= 2x^2 + 12x + 10$ No!

Try $\boxed{(2x + 5)(x + 2)}$ ← correct answer

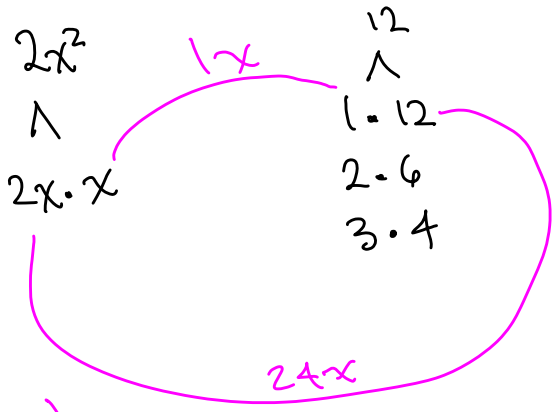
Check: $2x^2 + 4x + 5x + 10$

$= 2x^2 + 9x + 10$ ✓ Yes!!

Ex: $2x^2 - 25x + 12$

(+) same signs, sum of 25 x for middle term

$$(2x - 1)(x - 12)$$



Check:

$(2x-1)(x-12)$

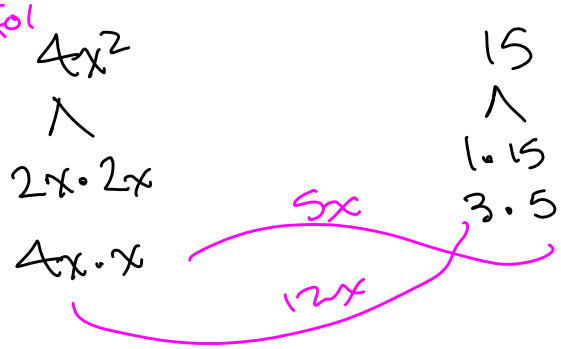
$= 2x^2 - 24x - 1x + 12$

$= 2x^2 - 25x + 12$ ✓

Ex: $4x^2 - 7x - 15$

(-) signs are opposite, want a difference of 7x for middle term

$$(4x + 5)(x - 3)$$



Check:

$4x^2 - 12x + 5x - 15$

$4x^2 - 7x - 15$ ✓