5.4 Cont'd (Difference of 2 Squares & Perfect Squares)

Note Title 4/5/2016

Try these: Facto

#1) $2x^3 - 72x$

#2) $4x^2 - 20x + 25$

#3) $y^2 + 81x^2$

#4) $x^2 + 6x + 9$

#5) $27a^2 - 12b^2$

#6) $x^5 - x$

#7) $x^4 - 13x^2 + 36$

#1) $2x^3 - 72x$

$= 2x(x^2 - 36)$

$\boxed{= 2x(x+6)(x-6)}$

Difference of 2 Squares:

$a^2 - b^2 = (a+b)(a-b)$

Check:

$(x+6)(x-6)$

$= x^2 - 6x + 6x - 36$

$= x^2 - 36$ ✓

Try this one: $(2x+12)(x^2-6x)$

Check: $2x^3 - 12x^2 + 12x^2 - 72x$

$= 2x^3 - 72x$ ✓

OK so far, but it can be factored more.

$(2x+12)(x^2-6x)$

$= 2(x+6)(x)(x-6)$

$= \boxed{2x(x+6)(x-6)}$

#2) $4x^2 - 20x + 25$

$(2x-5)(2x-5)$

$4x^2$ 25

$x \cdot 4x$ $1 \cdot 25$

$2x \cdot 2x$ $5 \cdot 5$

Check:

$4x^2 - 10x - 10x + 25$

$= 4x^2 - 20x + 25$ ✓

$\boxed{(2x-5)^2}$

Tip: If you are trying to factor a trinomial, and the 1st and last terms are positive squares, try factoring it as a perfect square before trying all the other trinomial possibilities

Ex: Factor.

$16x^2 - 56xy + 49y^2$

$16x^2$: $x \cdot 16x$, $2x \cdot 8x$, $4x \cdot 4x$

$49y^2$: $y \cdot 49y$, $7y \cdot 7y$

Try $(4x - 7y)^2$

Check: $(4x-7y)(4x-7y)$

$= 16x^2 - 28xy - 28xy + 49y^2$

$= 16x^2 - 56xy + 49y^2$ ✓ Yay!

$\boxed{(4x-7y)^2}$

Ex: $16x^2 - 120x + 81$

Try: $(4x - 9)^2$

Check: $(4x-9)(4x-9)$

$= 16x^2 - 36x - 36x + 81$

$= 16x^2 - 72x + 81$ Nope!

$16x^2$: $x \cdot 16x$, $2x \cdot 8x$, $4x \cdot 4x$

81: $1 \cdot 81$, $3 \cdot 27$, $9 \cdot 9$

$12x$ $108x$

(+) same signs
sum of 120x for middle

$12 + 108 = 120$...

$16x^2 - 120x + 81$

$= \boxed{(4x-27)(4x-3)}$

Check: $16x^2 - 12x - 108x + 81$

$= 16x^2 - 120x + 81$

#3) $y^2 + 81x^2$

$\boxed{\text{Prime}}$ (doesn't factor)

$\boxed{x^2 + \text{positive is always prime}}$

#4) $x^2 + 6x + 9$

$(x+3)(x+3)$

$\boxed{(x+3)^2}$

#5) $27a^2 - 12b^2$

$3(9a^2 - 4b^2)$

$= \boxed{3(3a+2b)(3a-2b)}$

#6) $x^5 - x$

$x(x^4 - 1)$

$= x((x^2)^2 - (1)^2)$

$= x(x^2+1)(x^2-1)$

$= \boxed{x(x^2+1)(x+1)(x-1)}$

$\leftarrow \left\{ \begin{array}{l} a^2 - b^2 \\ = (a+b)(a-b) \\ \text{with } a = x^2,\ b = 1 \end{array} \right.$

#7) $x^4 - 13x^2 + 36$

$= (x^2 - 9)(x^2 - 4)$

$= \boxed{(x+3)(x-3)(x+2)(x-2)}$

Check: $(x^2-9)(x^2-4)$

$= x^4 - 4x^2 - 9x^2 + 36$

$= x^4 - 13x^2 + 36$ ✓

5.5: Sum and Difference of Cubes

Sum and Difference of 2 Cubes Factorizations

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$
$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

Check: $(a+b)(a^2-ab+b^2)$
$= a^3 - \cancel{a^2b} + \cancel{ab^2} + \cancel{ba^2} - \cancel{ab^2} + b^3$
$= a^3+b^3$ ✓

Mnemonic Device (for remembering the $\pm$ signs)

M Match
O Opposite
P Positive (or Plus)

S Same
O Opposite
A Always
P Positive

Know your perfect cubes:

$$\begin{array}{r} {}^{6}49 \\ \times\ 7 \\ \hline 343 \end{array} \qquad \begin{array}{r} {}^{3}64 \\ \times\ 8 \\ \hline 512 \end{array} \qquad \begin{array}{r} 81 \\ \times\ 9 \\ \hline 729 \end{array}$$

$1^3=1$
$2^3=8$
$3^3=27$
$4^3=64$
$5^3=125$
$6^3=216$
$7^3=343$
$8^3=512$
$9^3=729$
$10^3=1000$

Example: Factor.

$$y^3 - 8$$

$$= y^3 - 2^3$$

$$= (y - 2)(y^2 + 2y + 2^2)$$

$$= \boxed{(y-2)(y^2+2y+4)}$$

using $a^3 - b^3$
$= (a-b)(a^2+ab+b^2)$
with $a = y$, $b = 2$

Check: $y^3 + 2y^2 + 4y$
$-2y^2 - 4y - 8$
$= y^3 - 8$ ✓

Ex: $x^3 + 27$

$$= x^3 + 3^3$$

$$= \boxed{(x+3)(x^2 - 3x + 9)}$$

Ex: $8x^3 + 125y^3$

$$= (2x)^3 + (5y)^3$$

$$= (2x + 5y)((2x)^2 - 2x(5y) + (5y)^2)$$

$$= \boxed{(2x+5y)(4x^2 - 10xy + 25y^2)}$$

common mistake: $(2x+5y)(2x^2 - 10xy + 5y^2)$

need to square the coefficient and the variable.

5.7 Solving Quadratic Equations by Factoring

A quadratic equation (in x) is an equation that can be written in the form $ax^2+bx+c=0$, where a, b, c are real numbers and $a \neq 0$.

$ax^2 + bx + c = 0$ is standard form for a quadratic eqn.

quadratic term   linear term   constant term

Zero Product Property (Zero Product Theorem)

If the product of 2 numbers is 0, then at least one of the numbers must be 0.

In other words, if $AB=0$, then $A=0$ or $B=0$.

Ex: Solve.

$$x^2 - 24 = 10x$$
$$\quad -10x \qquad -10x$$

Write in standard form:

$$x^2 - 10x - 24 = 0.$$

(−) signs are opposite
want a difference of $10x$

Factor it:

$$(x+2)(x-12)=0$$

Set each factor equal to 0:

$$x+2=0 \quad \text{OR} \quad x-12=0$$
$$-2 \quad -2 \qquad\qquad +12 \quad +12$$
$$x=-2 \qquad\qquad x=12$$

$$\boxed{\text{Solution Set: } \{-2, 12\}}$$

Remember:
* expressions get simplified (or factored)
* equations get solved

Check:
$(x+2)(x-12)$
$= x^2 - 12x + 2x - 24$
$x^2 - 10x - 24$

24
1·24
2·12
3·8
4·6