

5.7: Quadratic Equations (continued)

Note Title

4/7/2016

Recall: Steps for solving a quadratic equation

- 1) write in standard form $ax^2 + bx + c = 0$.
- 2) Factor the nonzero side.
- 3) Set each factor equal to 0. (Apply the Zero Product Property)
- 4) Solve the resulting linear equations.

Example: Solve.

$$-19x + x^2 = -60$$

$$x^2 - 19x + 60 = 0$$

$$(x - 4)(x - 15) = 0$$

$$x - 4 = 0$$

$$x = 4$$

or

$$x - 15 = 0$$

$$x = 15$$

Solution Set: $\{4, 15\}$

(+) same signs
sum of 19x

x^2	60
$x \cdot x$	1
	1 \cdot 60
	2 \cdot 30
	3 \cdot 20
	4 \cdot 15
	5 \cdot 12
	6 \cdot 10

Check:

$$x = 4 \quad -19x + x^2 = -60$$

$$-19(4) + 4^2 = -60$$

$$-76 + 16 = -60$$

$$-60 = -60 \quad \checkmark$$

$$x = 15 \quad -19(15) + (15)^2 = -60$$

$$-285 + 225 = -60$$

$$-60 = -60 \quad \checkmark$$

$$\begin{array}{r} 3 \\ 19 \\ \times 4 \\ \hline 76 \end{array}$$

$$\begin{array}{r} 2 \\ 15 \\ \times 15 \\ \hline 75 \\ 15 \\ \hline 225 \end{array}$$

Ex. Solve.

$$2x^3 - 14x^2 + 24x = 0$$

$$2x(x^2 - 7x + 12) = 0$$

$$2x(x - 3)(x - 4) = 0$$

$$2x = 0 \quad | \quad x - 3 = 0 \quad | \quad x - 4 = 0$$

$$x = 3$$

$$x = 4$$

$$\frac{2x}{2} = \frac{0}{2}$$

$$x = 0$$

Note: This is not a quadratic eqn.
This is a cubic eqn (degree = 3)

$$\begin{array}{c} x^2 \\ \wedge \\ x \cdot x \end{array}$$

$$\begin{array}{c} 12 \\ \wedge \\ 1 \cdot 12 \\ 2 \cdot 6 \\ \underline{3 \cdot 4} \end{array}$$

Sol'n Set: $\{0, 3, 4\}$

Check: $x=0 \quad 2(0)^3 - 14(0)^2 + 24(0) = 0$

$$0 = 0 \quad \checkmark$$

Could check other two solutions also.

Solve.

Ex. $-2x^2 + 70 = 4x$

$$-2x^2 - 4x + 70 = 0$$

$$-2(x^2 + 2x - 35) = 0$$

$$-2(x + 7)(x - 5) = 0$$

$$-2 = 0$$

False!

False for every x

This factor does not contribute any solutions

$$x + 7 = 0$$

$$x = -7$$

$$x - 5 = 0$$

$$x = 5$$

Solution Set: $\{-7, 5\}$

Scratchwork

Check:

$$(x+7)(x-5)$$

$$= x^2 - 5x + 7x - 35$$

$$= x^2 + 2x - 35$$

Ex: Solve.

$$x^2 = 9$$

$$x^2 - 9 = 0$$

$$(x-3)(x+3) = 0$$

$$x-3=0 \quad \text{or} \quad x+3=0$$

$$x=3$$

$$x=-3$$

Solution Set:

$$\{3, -3\}$$

Short hand:

$$\{\pm 3\}$$

"plus or minus 3"

Recall:
Difference of 2
Squares factorization:
 $a^2 - b^2 = (a+b)(a-b)$

Check: $x^2 = 9$

$$\underline{x=3} \quad (3)^2 = 9$$
$$9 = 9 \checkmark$$

$$\underline{x=-3} \quad (-3)^2 = 9$$
$$9 = 9 \checkmark$$

True

correct.

Ex:

$$7x + 5 = 6x^2$$

$$-6x^2 + 7x + 5 = 0$$

$$-1(6x^2 - 7x - 5) = 0$$

$$-1(2x+1)(3x-5) = 0$$

-1=0
False
for every
x

$$2x+1=0$$

$$2x = -1$$

$$\frac{2x}{2} = \frac{-1}{2}$$

$$x = -\frac{1}{2}$$

$$3x-5=0$$

$$3x = 5$$

$$\frac{3x}{3} = \frac{5}{3}$$

$$x = \frac{5}{3}$$

signs opposite
want difference of 7x

$$\begin{array}{r} 6x^2 \quad 5 \\ \wedge \quad \wedge \\ x-6x \quad 3x-1-5 \\ 2x-3x \quad 10x \end{array}$$

check: $(2x+1)(3x-5)$
 $6x^2 - 10x + 3x - 5$
 $6x^2 - 7x - 5 \checkmark$

Sol'n Set: $\{-\frac{1}{2}, \frac{5}{3}\}$

Previous example, alternate solution:

$$7x + 5 = 6x^2$$

$$0 = 6x^2 - 7x - 5$$

$$6x^2 - 7x - 5 = 0$$

$$(2x + 1)(3x - 5) = 0$$

$$2x + 1 = 0$$

$$3x - 5 = 0$$

$$2x = -1$$

$$3x = 5$$

$$\frac{2x}{2} = \frac{-1}{2}$$

$$\frac{3x}{3} = \frac{5}{3}$$

$$x = -\frac{1}{2}$$

$$x = \frac{5}{3}$$

$$\boxed{\text{Sol'n Set: } \left\{-\frac{1}{2}, \frac{5}{3}\right\}}$$

Ex: $x^3 - 6x^2 = -9x$

$$x^3 - 6x^2 + 9x = 0$$

$$x(x^2 - 6x + 9) = 0$$

$$x(x-3)(x-3) = 0$$

$$x=0 \mid x-3=0 \mid x-3=0$$

$$x=3 \mid x=3$$

$$\text{Solution Set: } \boxed{\{0, 3\}}$$

Note: $x=0$ is a solution!

check: $x^3 - 6x^2 = -9x$

$$x=0 \mid 0^3 - 6(0)^2 = -9(0)$$

$$0 = 0 \text{ True } \checkmark$$

5.8: Applications of Quadratic Equations

(Word problems!)

(This section will be on Test 4)

Ex: The length of a rectangle is 6" more than four times its width. The area of the rectangle is 70 square inches. Find the length and width.

length: $4x+6$
width: x

length $\xrightarrow{\text{compares to}}$ width x

$$\text{Area} = (\text{length})(\text{width})$$

$$70 = (4x+6)(x)$$

Rearrange:

$$x(4x+6) = 70$$

$$4x^2 + 6x = 70$$

$$4x^2 + 6x - 70 = 0$$

$$2(2x^2 + 3x - 35) = 0$$

(-) signs opposite difference of 3

$$2(2x - 7)(x + 5) = 0$$

$2=0$	$2x-7=0$	$x+5=0$
False!	$2x=7$	$x=-5$
	$\frac{2x}{2} = \frac{7}{2}$	
	$x = \frac{7}{2}$	

Sol'n Set for this eqn: $\{\frac{7}{2}, -5\}$

$$\begin{array}{cc} 2x^2 & 35 \\ \uparrow & \uparrow \\ 2x \cdot 7 & 1 \cdot 35 \\ \swarrow \quad \searrow & \\ 10x & 5 \cdot 7 \end{array}$$

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Previous example cont'd:

Our 2 solutions are $x = \frac{7}{2}$, $x = -5$

What does x represent? width.

Throw out -5: A negative number does not make sense for a dimension.

The only solution that makes sense is $\frac{7}{2}$.

width: $x = \frac{7}{2} = 3\frac{1}{2}$ inches

length: $4x + 6$

$$x = \frac{7}{2} \Rightarrow 4\left(\frac{7}{2}\right) + 6$$
$$= \frac{28}{2} + 6 = 14 + 6 = 20 \text{ inches}$$

The width is $3\frac{1}{2}''$ and the length is $20''$.

(write answer in a complete sentence.)

Check: 1st sentence:

$$4 \text{ times width: } 4\left(\frac{7}{2}''\right) = \frac{28}{2}'' = 14''$$

6'' more: $20''$ ✓ 1st sentence checks

2nd sentence: Is the area 70 in^2 ?

$$20'' \left(3\frac{1}{2}''\right)$$
$$= \underline{20} \text{ in} \left(\frac{7}{2} \text{ in}\right) = \frac{140}{2} \text{ in}^2 = 70 \text{ in}^2$$