5.8: Applications of Quadratic Eqns (cont'd)

Note Title 4/14/2016

Example: The height of a triangle is 3 feet less than $\underbrace{\text{five times its base}}_{}^{5x}$. The area of the triangle is 13 ft^2. Find the base and height.

$ft^2 = (ft)(ft) =$ square feet

base: x

height: $5x-3$

height $\xrightarrow{\text{compare to}}$ base

x

Equivalent

Area of triangle $= \frac{(\text{base})(\text{ht})}{2}$

$$\text{Area of triangle} = \frac{1}{2}(\text{base})(\text{height})$$

$$13 = \frac{1}{2}(x)(5x-3)$$

Multiply both sides by 2:

$$(2)\,13 = (2)\frac{1}{2}x(5x-3)$$

$$26 = x(5x-3)$$

$$26 = 5x^2 - 3x$$

write in standard form (with one side 0):

$$0 = 5x^2 - 3x - 26$$

(-) opp. signs

want a difference of $3x$

$$5x^2 - 3x - 26 = 0$$

Factor:

$$(5x - 13)(x + 2) = 0$$

scratch

$5x^2$	26
$x \cdot 5x$	$1 \cdot 26$
	$2 \cdot 13$

$10x$

$13x$

$5x - 13 = 0$

+13 +13

$5x = 13$

$x = \frac{13}{5}$

$x + 2 = 0$

$x = -2$

Throw out -2. A negative number does not make sense for a dimension

~~See next page~~

Previous example cont'd:

So our only solution that makes sense is $x = \frac{13}{5}$

base: $x = \frac{13}{5} = 2\frac{3}{5}$ feet

height: $5x - 3$

Substitute $x = \frac{13}{5}$: $\cancel{5}^{1}\left(\frac{13}{\cancel{5}_{1}}\right) - 3 = 13 - 3 = 10$ ft

$\boxed{\text{The base is } 2\frac{3}{5} \text{ ft and the height is } 10 \text{ ft.}}$

Check it: 1st sentence: 5 times base $= 5\left(2\frac{3}{5}\right)$

$= 5\left(\frac{13}{5}\right) = 13$

3 ft less: 10 ft same as height ✓

2nd sentence: Is area 13 ft^2?

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$

$$= \frac{1}{2}\left(\frac{13}{5} \text{ ft}\right)(10 \text{ ft})$$

$$= \frac{13}{10} \text{ ft}\,(10 \text{ ft})$$

$$= \frac{13}{\cancel{10}_{1}} \cdot \cancel{10}^{1} \text{ ft} \cdot \text{ft} = 13 \text{ ft}^2$$

✓ OK

Example: The hypotenuse of a right triangle is 2" longer than the $\overbrace{\text{longer leg}}^{2x+4}$. The longer leg is 4" longer than $\overbrace{\text{twice the shorter leg}}^{2x}$. Find the lengths of all three sides.

hypotenuse: $2x+4+2 = 2x+6$

shorter leg: x

longer leg: $2x+4$

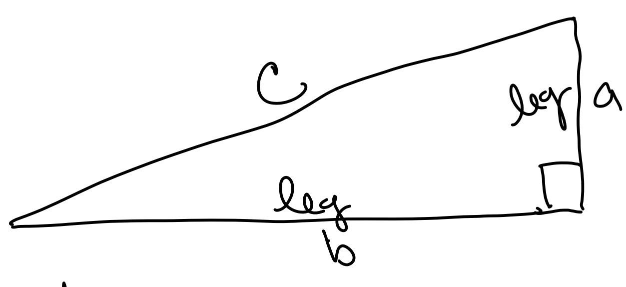
hypotenuse $\xrightarrow[\text{to}]{\text{compare}}$ longer leg ($2x+4$) $\xrightarrow[\text{to}]{\text{compare}}$ short leg (x)

Recall: right triangle: has a $90°$ angle

Hypotenuse = longest side in a right triangle

Pythagorean Theorem:

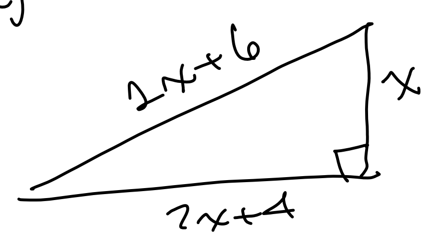
If a and b are the legs of a right triangle, and if c is the hypotenuse, then $a^2+b^2=c^2$.



$a^2+b^2=c^2$

$$\begin{pmatrix}\text{long}\\\text{leg}\end{pmatrix}^2 + \begin{pmatrix}\text{short}\\\text{leg}\end{pmatrix}^2 = (\text{hypotenuse})^2$$

$$(2x+4)^2 + (x)^2 = (2x+6)^2$$



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Previous example cont'd:

$$(2x+4)^2 + (x)^2 = (2x+6)^2$$

$$(2x+4)(2x+4) + x^2 = (2x+6)(2x+6)$$

$$4x^2 + 8x + 8x + 16 + x^2 = 4x^2 + 12x + 12x + 36$$

$$4x^2 + 16x + 16 + x^2 = 4x^2 + 24x + 36$$

$$5x^2 + 16x + 16 = 4x^2 + 24x + 36$$

$$-4x^2 \quad -24x \quad -36 \qquad -4x^2 \quad -24x \quad -36$$

$$x^2 - 8x - 20 = 0$$

$$(x+2)(x-10) = 0$$

check:
$(x+2)(x-10)$
$= x^2 - 10x + 2x - 20$
$= x^2 - 8x - 20$ ✓

$x + 2 = 0$ | $x - 10 = 0$

$x = -2$ | $x = 10$

Throw out!
(can't use a negative for a dimension)

shorter leg: $x = 10$ in

longer leg: $2x + 4$

$x = 10 \Rightarrow 2(10) + 4$
$= 20 + 4 = 24''$

hypotenuse: $2x + 6$

$x = 10 \Rightarrow 2(10) + 6 = 26''$

The hypotenuse is 26'' and the legs are 10'' and 24''.

OR

The sides are 10'', 24'' and 26''.

Check: 1st check that sentence #1 and sentence #2 are true (they are)

Check that it's a right triangle:

Is $a^2+b^2=c^2$?

$10^2+24^2 \overset{?}{=} 26^2$

$100+576=676$

$676=676$ ✓

$$\begin{array}{r} 24 \\ \times 24 \\ \hline 96 \\ 48 \\ \hline 576 \end{array} \qquad \begin{array}{r} 26 \\ \times 26 \\ \hline 156 \\ 52 \\ \hline 676 \end{array}$$

Ex: Find 2 consecutive integers whose product is 27 more than 5 times the larger integer.

1st integer: x

2nd integer: $x+1$

Note:
Consecutive even integers:
$x, x+2, x+4$
consecutive odd integers
$x, x+2, x+4$

Product $= 5$ (larger integer) $+ 27$

$\underbrace{x(x+1)}_{\text{product}} = 5(x+1)+27$

$x^2+1x = 5x+5+27$

$x^2+x = 5x+32$
$-5x-32 \quad -5x \quad -32$

$x^2-4x-32=0$

$(x-8)(x+4)=0$

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Previous example cont'd:

$(x-8)(x+4)=0$

$x-8=0$ or $x+4=0$

$x=8$ | $x=-4$

Both of these values for x are acceptable.

$x=8$ | 1st integer: $x=8$

2nd integer: $x+1$

$=8+1=9$

The integers are 8 and 9.

$x=-4$ | 1st integer: $x=-4$

2nd integer: $x+1$

$=-4+1$

$=-3$

Here, the integers are -4 and -3.

The pair of integers is either 8, 9 or -4, -3.

Check: 8, 9

are they consecutive? Yes

Product: $8(9)=72$

5 times larger: $5(9)=45$

27 more: $45 + 27 = 72$ ✓

same

Check: $-4, -3$

are they consecutive? Yes

Product: $-4(-3)=12$

5 times larger: $5(-3)=-15$

27 more: $-15+27=12$

same ✓

6.1: Simplifying Rational expressions

<u>Recall</u>: rational number: can be written as $\frac{a}{b}$ where a and b are integers.

A rational number: ratio of 2 integers

Examples of rational numbers:

$$\frac{2}{3},\ -\frac{5}{3},\ \frac{1}{4},\ 2,\ 0.13,\ -7$$

$$2 = \frac{2}{1},\quad 0.13 = \frac{13}{100},\quad -7 = \frac{-7}{1}$$

<u>Not rational (irrational numbers)</u>

$$\pi,\ \sqrt{3},\ \sqrt{2}$$

<u>Rational expression</u>: ratio of 2 polynomials.

<u>Ex</u>: $\frac{x^2+5}{x-9}$, $\frac{x^2-6x+4}{3x^2-7}$, $\frac{1}{x-7}$

<u>Simplifying rational expressions</u>:

Ex: $\frac{10}{24} = \frac{\cancel{2}^{1}\cdot 5}{\cancel{2}_{1}\cdot 12} = \frac{\cancel{2}}{\cancel{2}}\cdot\frac{5}{12} = \boxed{\frac{5}{12}}$

<u>Ex</u>: Simplify.

$$\frac{x^2+8x+15}{x^2+5x+6} = \frac{\cancel{(x+3)}(x+5)}{(x+2)\cancel{(x+3)}} = \boxed{\frac{x+5}{x+2}}$$

Ex: Simplify.

$$\frac{x-3}{x^2-7x+12}$$

$$= \frac{\cancel{x-3}^{\,1}}{\cancel{(x-3)}_{1}(x-4)} = \boxed{\frac{1}{x-4}}$$

Note: $\frac{8}{24} = \frac{1}{3}$ $\qquad \frac{8}{24} = \frac{\cancel{8}^{\,1}}{\cancel{8}_{1}\cdot 3} = \boxed{\frac{1}{3}}$

Quickie Preview of 6.2, for those who need to use their weekends to get ahead:

6.2: Multiplying & Dividing Rational Expressions

Ex: $\frac{2}{3}\cdot\frac{4}{5} = \frac{8}{15}$

$$\frac{\cancel{16}^{\,1}}{\cancel{15}_{5}}\cdot\frac{\cancel{21}^{\,7}}{\cancel{32}_{2}} = \boxed{\frac{7}{10}}$$

Ex: $\frac{x^2+2x}{x^2-4}\cdot\frac{x^2-4x+4}{x^2-x}$

$$= \frac{\cancel{x}^{\,1}\cancel{(x+2)}}{\cancel{(x+2)}_{1}\cancel{(x-2)}_{1}}\cdot\frac{\cancel{(x-2)}^{\,1}(x-2)}{\cancel{x}_{1}(x-1)} = \boxed{\frac{x-2}{x-1}}$$