

6.1 and 6.2 Cont'd:

Note Title

4/21/2016

6.1: Simplifying Rational Expressions

Ex: Simplify: $\frac{x^3 - 25x}{x^2 + x - 30}$

$$= \frac{x(x^2 - 25)}{(x - 5)(x + 6)}$$

$$= \frac{x(x+5)(x-5)}{(x-5)(x+6)}$$

$$= \boxed{\frac{x(x+5)}{x+6}}$$

This equals sign is not true for $x=5, x=-6$

Note: this fraction is undefined for $x=5, x=-6$

Scratchwork

$$\begin{aligned} x^3 - 25x &= x(x^2 - 25) \\ &= x(x+5)(x-5) \end{aligned}$$

$$x^2 + x - 30 \quad \text{opp signs}$$

$$(x - 5)(x + 6)$$

check
 $x^2 + 6x - 5x - 30$
 $x^2 + x - 30$ ✓

Note:
Restrictions: $x \neq 5$
 $x \neq -6$

Example: simplify:

$$\frac{8x^2 + 10x + 3}{8x^2 + 2x - 3}$$

$$= \frac{(2x+1)(4x+3)}{(2x-1)(4x+3)}$$

$$= \boxed{\frac{2x+1}{2x-1}}$$

Scratchwork

Numerator:

$$8x^2 + 10x + 3$$

$$(2x+1)(4x+3)$$

check

$$8x^2 + 6x + 4x + 3$$

10x ✓

Denominator

$$8x^2 + 2x - 3$$

$$(2x-1)(4x+3)$$

$$8x^2 + 6x - 4x - 3$$

$$= 8x^2 + 2x - 3$$

(+) same signs sum of 10x

$$\begin{array}{r} 8x^2 \quad 3 \\ \wedge \quad \wedge \\ (2x+1)(4x+3) \\ x \cdot 8x \quad 1 \cdot 3 \\ 2x \cdot 4x \end{array}$$

$$6x$$

(-) opp. signs diff of 2x

$$\begin{array}{r} 8x^2 \quad 3 \\ \wedge \quad \wedge \\ (2x-1)(4x+3) \\ x \cdot 8x \quad 1 \cdot 3 \\ 2x \cdot 4x \end{array}$$

4x 6x

Ex: Simplify.

$$\frac{5x^4y^2 - 15x^3y^3}{10x^6y^3 + 40x^5y^4}$$

$$= \frac{\cancel{5}x^{\cancel{3}}y^{\cancel{2}}(x-3y)}{\cancel{10}x^{\cancel{5}}y^{\cancel{3}}(x+4y)}$$

$$= \frac{1(x-3y)}{2x^2y(x+4y)}$$

$$\boxed{\frac{x-3y}{2x^2y(x+4y)}}$$

Also correct:

$$\boxed{\frac{x-3y}{2x^3y + 8x^2y^2}}$$

Ex: Simplify:

$$\frac{x-2}{4-x^2}$$

$$= \frac{x-2}{-x^2+4}$$

$$= \frac{x-2}{-1(x^2-4)}$$

$$= \frac{\cancel{x-2}}{-1(\cancel{x+2})(x-2)}$$

$$= \frac{1}{-1(x+2)}$$

$$= \boxed{-\frac{1}{x+2}}$$

Note:

$$a-b = -1(b-a)$$

Note:

$$\left. \begin{aligned} \frac{-2}{8/2} &= -\frac{1}{4} \\ \frac{-1/2}{1/2} &= -\frac{1}{4} \\ \frac{-1/2}{1/2} &= -\frac{1}{4} \end{aligned} \right\} \text{all are equal}$$

Note:

$$\frac{\cancel{x-3}}{(x+3)\cancel{(x-3)}} = \boxed{\frac{1}{x+3}}$$

$$\frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}} = \frac{x+3}{1} = \boxed{x+3}$$

Note:

previous problem:

$$\frac{x-2}{4-x^2} = \frac{x-2}{(2+x)(2-x)} = \frac{x-2}{(2+x)(-1)(-2+x)}$$

$$= \frac{\cancel{x-2}}{(2+x)(-1)\cancel{(x-2)}} = \frac{1}{-1(2+x)}$$

$$= \boxed{-\frac{1}{2+x}}$$

$$= \boxed{-\frac{1}{x+2}}$$

6.2: Multiplying and Dividing Rational Expressions

Ex: $\frac{4}{7} \cdot \frac{2}{3} = \frac{4 \cdot 2}{7 \cdot 3} = \boxed{\frac{8}{21}}$

$$\frac{\overset{3}{\cancel{24}}}{\underset{5}{\cancel{35}}} \cdot \frac{\overset{3}{\cancel{21}}}{\underset{8}{\cancel{64}}} = \boxed{\frac{9}{40}}$$

$$\frac{18}{24} \div \frac{15}{8} = \frac{\overset{6}{\cancel{18}}}{\underset{3}{\cancel{24}}} \cdot \frac{\overset{8}{\cancel{8}}}{\underset{5}{\cancel{15}}} = \frac{\overset{2}{\cancel{6}}}{\underset{5}{\cancel{15}}} = \boxed{\frac{2}{5}}$$

Ex: simplify.

$$\underline{\text{Ex:}} \quad \frac{3x^2y}{4z} \cdot \frac{16z^5}{xy^3} = \frac{12x^2y z^5}{xy^3z} = \boxed{\frac{12x z^4}{y^2}}$$

$$\underline{\text{Ex:}} \quad \frac{x^3 - 9x}{x^2 - 3x - 18} \cdot \frac{x^2 - x - 30}{x^2 - 7x + 12}$$
$$\frac{x(x+3)(x-3)}{(x-6)(x+3)} \cdot \frac{(x-6)(x+5)}{(x-3)(x-4)}$$
$$= \boxed{\frac{x(x+5)}{x-4}}$$

Scratchwork
1st numerator:
 $x^3 - 9x$
 $x(x^2 - 9)$
 $x(x+3)(x-3)$

$$\underline{\text{Ex:}} \quad (x+2) \cdot \frac{5}{4x^2 - 16}$$
$$= \frac{x+2}{1} \cdot \frac{5}{4(x^2 - 4)}$$
$$= \frac{x+2}{1} \cdot \frac{5}{4(x+2)(x-2)}$$
$$= \boxed{\frac{5}{4(x-2)}}$$

Ex.: $\frac{x^2 + 2x}{x+5} \div \frac{4-x^2}{3x-6}$

$$= \frac{x^2 + 2x}{x+5} \cdot \frac{3x-6}{-x^2+4}$$

[change to multiplication]

$$= \frac{x(x+2)}{x+5} \cdot \frac{3(x-2)}{-1(x^2-4)}$$

$$= \frac{x(x+2)}{x+5} \cdot \frac{3(x-2)}{-1(x+2)(x-2)}$$

$$= \frac{3x}{-1(x+5)}$$

$$= -\frac{3x}{x+5}$$

Important: you can only cancel factors, not terms:
Can I cancel x here? No!

Why not?

$$\frac{x}{x^2+5x} = \frac{x}{x(x+5)} = \frac{\cancel{x}}{\cancel{x}} \cdot \frac{1}{x+5}$$

Try this w/ $x=3$:

$$\begin{aligned} \frac{x}{x^2+5x} &= \frac{3}{3^2+5(3)} = \frac{3}{24} = \frac{1}{8} \\ \frac{1}{x+5} &= \frac{1}{3+5} = \frac{1}{8} \end{aligned}$$

$$x=3 \Rightarrow \frac{3}{3+5} = \frac{3}{8} \neq \frac{1}{5} \text{ or } \frac{1}{1+5} = \frac{1}{6}$$

not equal