

6.3: Addition and Subtraction of

Note Title

Rational Expressions

4/26/2016

Ex: Multiplication: $\frac{3}{5} \cdot \frac{4}{5} = \frac{3(4)}{5(5)} = \boxed{\frac{12}{25}}$

Addition:
(same denominators) $\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \boxed{\frac{7}{5}}$

Addition
(denominators not the same) : $\frac{3}{5} + \frac{4}{7}$
 $= \frac{3}{5} \left(\frac{7}{7}\right) + \frac{4}{7} \left(\frac{5}{5}\right)$
 $= \frac{21}{35} + \frac{20}{35} = \boxed{\frac{41}{35}}$

Multiplication
(different denominators) : $\frac{3}{5} \cdot \frac{4}{7} = \frac{3(4)}{5(7)} = \boxed{\frac{12}{35}}$

(we need common denominators to add/subtract,
but not to multiply)

Example: Simplify. (add or subtract)

$$\frac{1}{6x} + \frac{7}{6x} = \frac{1+7}{6x} = \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{6}x}} = \boxed{\frac{4}{3x}}$$

Ex:

$$\frac{3}{x+5} - \frac{10}{x+5}$$

$$= \frac{3-10}{x+5}$$

$$= \boxed{\frac{-7}{x+5}}$$

both
OK.

$$= \boxed{-\frac{7}{x+5}}$$

Ex:

$$\frac{4x}{x+1} + \frac{x+3}{x+1}$$

$$= \frac{4x+x+3}{x+1}$$

$$= \boxed{\frac{5x+3}{x+1}}$$

Note:

$$\frac{4x}{x+1} \cdot \frac{x+3}{x+1}$$

$$= \boxed{\frac{4x(x+3)}{(x+1)^2}}$$

$$= \boxed{\frac{4x^2 + 12x}{x^2 + 2x + 1}}$$

easier to
leave in
Factored
form
(and more
useful for
graphing,
etc)

Ex:

$$\frac{3y-5}{y+4} - \frac{5y-7}{y+4}$$

$$= \frac{3y-5-(5y-7)}{y+4}$$

$$= \frac{3y-5-5y+7}{y+4}$$

$$= \boxed{\frac{-2y+2}{y+4}}$$

$$= \boxed{\frac{-2(y-1)}{y+4}}$$

Both
OK

Ex.:

$$\frac{x^2 + 1}{x^2 - x - 2} - \frac{3x + 4}{x^2 - x - 2}$$

$$= \frac{x^2 + 1 - 3x - 4}{x^2 - x - 2}$$

=

$$\boxed{\frac{x^2 - 3x - 3}{x^2 - x - 2}}$$

$$= \boxed{\frac{x^2 - 3x - 3}{(x-2)(x+1)}}$$

$$\begin{array}{r} x^2 \\ \downarrow \\ x-x \end{array} \quad \begin{array}{r} 3 \\ \downarrow \\ 1 \cdot 3 \end{array}$$

Ex.:

$$\frac{2x+1}{x^2-1} + \frac{x+2}{x^2-1}$$

$$= \frac{2x+1+x+2}{x^2-1} = \frac{3x+3}{x^2-1}$$

$$= \frac{3(x+1)}{(x+1)(x-1)} = \boxed{\frac{3}{x-1}}$$

Ex: $\frac{x+5}{x-3} + \frac{2x-7}{3-x}$

Need to factor -1 out of one of these denominators.

Note:

$$a-b = -(b-a) \\ = -b+a$$

$$\frac{x+5}{x-3} + \frac{2x-7}{-1(x-3)}$$

$$= \frac{x+5}{x-3} - \frac{2x-7}{x-3}$$

$$= \frac{x+5}{x-3} + \frac{-2x+7}{x-3}$$

$$= \frac{x+5-2x+7}{x-3}$$

$$= \boxed{\frac{-x+12}{x-3}}$$

Note:

$$\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$$

OR

$$\frac{x+5-(2x-7)}{x-3}$$

$$x-3$$

$$= \frac{x+5-2x+7}{x-3}$$

$$= \frac{-x+12}{x-3}$$

Examples that do not have
common denominators:

LCD: 60

(Least Common Denominator)

Ex: $\frac{7x}{30} - \frac{11x}{12}$

$$= \frac{7x}{30} \left(\frac{2}{2} \right) - \frac{11x}{12} \left(\frac{5}{5} \right)$$

$$= \frac{14x}{60} - \frac{55x}{60}$$

$$= \boxed{\frac{-41x}{60}} = \boxed{-\frac{41x}{60}}$$

Ex: $\frac{5y}{6x^2} - \frac{y}{4x}$

$$\frac{5y}{6x^2} \left(\frac{2}{2} \right) + \frac{-y}{4x} \left(\frac{3x}{3x} \right)$$

$$= \frac{10y}{12x^2} + \frac{-3xy}{12x^2}$$

$$= \boxed{\frac{10y - 3xy}{12x^2}} = \boxed{\frac{y(10 - 3x)}{12x^2}}$$

LCD: $12x^2$

(choose the biggest power of each variable that appears in any of the denominators)

Ex: $\frac{2}{xy^4} + \frac{3}{x^2y^2}$

LCD: x^2y^4

$$= \frac{2}{xy^4} \left(\frac{x}{x} \right) + \frac{3}{x^2y^2} \left(\frac{y^2}{y^2} \right)$$

$$= \frac{2x}{x^2y^4} + \frac{3y^2}{x^2y^4}$$

$$= \boxed{\frac{2x + 3y^2}{x^2y^4}}$$

Ex: $\frac{3}{x} + \frac{5}{y^2} - \frac{8}{z}$

LCD: xy^2z

$$= \frac{3}{x} \left(\frac{y^2z}{y^2z} \right) + \frac{5}{y^2} \left(\frac{xz}{xz} \right) - \frac{8}{z} \left(\frac{xy^2}{xy^2} \right)$$

$$= \frac{3y^2z}{xy^2z} + \frac{5xz}{xy^2z} - \frac{8xy^2}{xy^2z}$$

$$= \boxed{\frac{3y^2z + 5xz - 8xy^2}{xy^2z}}$$

Ex:

$$\frac{3}{x-3} - \frac{6}{x+5}$$

$$\text{LCD: } (x-3)(x+5)$$

$$\frac{3}{x-3} \left(\frac{x+5}{x+5} \right) + \frac{-6}{x+5} \left(\frac{x-3}{x-3} \right)$$

$$= \frac{3x+15-6x+18}{(x+5)(x-3)} = \boxed{\frac{-3x+33}{(x+5)(x-3)}}$$

$$= \boxed{\frac{-3(x-11)}{(x+5)(x-3)}}$$

↑
okay too

Never write:

$$\frac{-3x+33}{x-2}$$

$$\boxed{\frac{-3x+33}{x^2+2x-15}}$$

is this supposed to be

$$\frac{-3x+33}{x-2} \quad \text{or} \quad -\frac{3x+33}{x-2}$$

$$\uparrow \frac{-3x-33}{x-2}$$

↑
not equal!

Ex: $\frac{3x}{3x-2} - \frac{5}{4x}$

LCD: $4x(3x-2)$

$$\frac{3x}{3x-2} \left(\frac{4x}{4x} \right) + \frac{-5}{4x} \left(\frac{3x-2}{3x-2} \right)$$

$$= \boxed{\frac{12x^2 - 15x + 10}{4x(3x-2)}}$$

$12x^2$	10
$\wedge$	$\wedge$
$x \cdot 12x$	$1 \cdot 10$
$2x \cdot 6x$	$2 \cdot 5$
$3x \cdot 4x$	

Ex: $\frac{4x+1}{x^2-3x} - \frac{3x}{x^2-x-6}$

$$= \frac{4x+1}{x(x-3)} + \frac{-3x}{(x-3)(x+2)}$$

[Factor all the denominators]

$$= \frac{4x+1}{x(x-3)} \left(\frac{x+2}{x+2} \right) + \frac{-3x}{(x-3)(x+2)} \left(\frac{x}{x} \right) \quad \text{LCD: } x(x-3)(x+2)$$

$$= \frac{4x^2 + 8x + x + 2 - 3x^2}{x(x-3)(x+2)}$$

$$= \frac{4x^2 + 9x + 2 - 3x^2}{x(x-3)(x+2)} = \boxed{\frac{x^2 + 9x + 2}{x(x-3)(x+2)}}$$

$$\text{Ex: } \frac{2x-1}{x^2+x-6} - \frac{x+2}{x^2+5x+6}$$

$$= \frac{2x-1}{(x+3)(x-2)} + \frac{-1(x+2)}{(x+2)(x+3)}$$

$$= \frac{2x-1}{(x+3)(x-2)} \left(\frac{x+2}{x+2} \right) + \frac{-1(x+2)}{(x+2)(x+3)} \left(\frac{x-2}{x-2} \right)$$

$$= \frac{2x^2 + 3x - 2 - 1(x^2 - 4)}{(x+3)(x-2)(x+2)}$$

$$= \frac{2x^2 + 3x - 2 - x^2 + 4}{(x+3)(x-2)(x+2)}$$

$$= \frac{x^2 + 3x + 2}{(x+3)(x-2)(x+2)} = \frac{\cancel{(x+2)}(x+1)}{(x+3)(x-2)\cancel{(x+2)}}$$

$$= \boxed{\frac{x+1}{(x+3)(x-2)}}$$

check

$$\begin{aligned} & (x+3)(x-2) \\ &= x^2 - 2x + 3x - 6 \\ &= x^2 + x - 6 \quad \checkmark \\ & (x+2)(x+3) \\ &= x^2 + 2x + 3x + 6 \\ &= x^2 + 5x + 6 \quad \checkmark \end{aligned}$$

LCD:

$$(x+3)(x-2)(x+2)$$