Note Title 4/28/2016

6.2 #7

$$\frac{x+1}{x^2-9} \div \frac{2x+2}{x+3}$$

$$= \frac{x+1}{x^2-9} \cdot \frac{x+3}{2x+2}$$

$$= \frac{\cancel{x+1}}{\cancel{(x+3)}(x-3)} \cdot \frac{\cancel{x+3}}{2\cancel{(x+1)}} = \boxed{\frac{1}{2(x-3)}}$$

6.4: Solving Rational Equations

Recall: To solve an equation means to find all the values of the variable that make it true.

Ex: Solve.

$$\frac{x}{5} + \frac{1}{5} = \frac{4}{5}$$

Solving by inspection: $x = 3$

Solution set: $\{3\}$

Check: $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

$\frac{4}{5} = \frac{4}{5}$ ✓ True

Ex: Solve.

$$\frac{x}{8} = \frac{7}{8}$$

By inspection: $x = 7$

Sol'n set: $\{7\}$

Notice: If 2 fractions are equal, and they have the same denominators, then their numerators must be equal.

Ex: Solve.

$$\frac{x}{5} - \frac{4x}{3} = \frac{1}{15} \qquad \text{LCD: } 15$$

$$\frac{x}{5}\left(\frac{3}{3}\right) + \frac{-4x}{3}\left(\frac{5}{5}\right) = \frac{1}{15}$$

$$\frac{3x}{15} + \frac{-20x}{15} = \frac{1}{15}$$

$$\frac{-17x}{15} = \frac{1}{15}$$

$$\frac{\cancel{15}}{1}\left(\frac{-17x}{\cancel{15}}\right) = \left(\frac{1}{\cancel{15}}\right)\frac{\cancel{15}}{1}$$

$$-17x = 1$$

$$\frac{-17x}{-17} = \frac{1}{-17}$$

$$x = -\frac{1}{17}$$

$$\boxed{\left\{-\frac{1}{17}\right\}}$$

Check it:

$$\frac{x}{5} - \frac{4x}{3} = \frac{1}{15}$$

$$\frac{-\frac{1}{17}}{5} - \frac{4\left(-\frac{1}{17}\right)}{3} = \frac{1}{15}$$

$$-\frac{1}{17}\cdot\frac{1}{5} + \frac{4}{17}\cdot\frac{1}{3} = \frac{1}{15}$$

$$-\frac{1}{17(5)}\left(\frac{3}{3}\right) + \frac{4}{17(3)}\left(\frac{5}{5}\right) = \frac{1}{15}$$

$$\frac{-3}{17\cdot 5\cdot 3} + \frac{20}{17\cdot 3\cdot 5} = \frac{1}{15}$$

$$\frac{\cancel{17}^{\,1}}{\cancel{17}_{\,1}\cdot 3\cdot 5} = \frac{1}{15}$$

$$\frac{1}{15} = \frac{1}{15} \quad ✓$$

Ex: Solve

$$\frac{2x-1}{2} - \frac{4x+3}{3} = \frac{x+5}{6} \qquad \text{LCD: } 6$$

$$\frac{2x-1}{2}\left(\frac{3}{3}\right) + \frac{-4x-3}{3}\left(\frac{2}{2}\right) = \frac{x+5}{6}$$

$$\frac{6x-3}{6} + \frac{-8x-6}{6} = \frac{x+5}{6}$$

$$\frac{6x-3-8x-6}{6} = \frac{x+5}{6}$$

$$\frac{-2x-9}{6} = \frac{x+5}{6}$$

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Previous example cont'd:

$$\frac{-2x-9}{6} = \frac{x+5}{6}$$

$$\left(\frac{\cancel{6}}{1}\right)\left(\frac{-2x-9}{\cancel{6}}\right) = \frac{x+5}{\cancel{6}}\left(\frac{\cancel{6}}{1}\right)$$

$$-2x-9 = x+5$$

$$-x \qquad -x$$

$$-3x-9 = 5$$

$$+9 \qquad +9$$

$$-3x = 14$$

$$\frac{-3x}{-3} = \frac{14}{-3}$$

$$x = -\frac{14}{3}$$

$$\boxed{\left\{-\frac{14}{3}\right\}}$$

Ex: Solve.

$$\frac{1}{3x} - \frac{1}{6} = \frac{5}{x}$$

LCD: $6x$

Restriction

$x \neq 0$

$$\frac{1}{3x}\left(\frac{2}{2}\right) + \frac{-1}{6}\left(\frac{x}{x}\right) = \frac{5}{x}\left(\frac{6}{6}\right)$$

$$\frac{2}{6x} + \frac{-x}{6x} = \frac{30}{6x}$$

$$2-x = 30$$

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Previous example cont'd

$2-x=30$

$\;\;{-2}\qquad\;\;{-2}$

$-x=28$

$\frac{-x}{-1}=\frac{28}{-1}$

$x=-28$

Sol'n Set:

$\boxed{\{-28\}}$

Solve.

Ex: $\frac{2x}{x-3}-\frac{1}{2}=\frac{6}{x-3}$

LCD: $2(x-3)$

$\frac{2x}{x-3}\left(\frac{2}{2}\right)+\frac{-1}{2}\left(\frac{x-3}{x-3}\right)=\frac{6}{x-3}\left(\frac{2}{2}\right)$

Note: Restrictions

$x\neq 3$

$\frac{4x}{2(x-3)}+\frac{-x+3}{2(x-3)}=\frac{12}{2(x-3)}$

$\frac{4x-x+3}{2(x-3)}=\frac{12}{2(x-3)}$

$\frac{3x+3}{2(x-3)}=\frac{12}{2(x-3)}$

$3x+3=12$

$3x=9$

$\frac{3x}{3}=\frac{9}{3}$

$x=3$

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Previous example cont'd

Check our answer in the original problem:

$$\frac{2x}{x-3} - \frac{1}{2} = \frac{6}{x-3}$$

$x=3 \Longrightarrow$ $$\frac{2(3)}{3-3} - \frac{1}{2} = \frac{6}{3-3}$$

$$\frac{6}{0} - \frac{1}{2} = \frac{6}{0}$$

Division by 0 is undefined!

We cannot have a 0 denominator!

$x=3$ does <u>not</u> check in the original eqn.

(it doesn't give a true statement)

(it doesn't make the original eqn true)

Throw out $x=3$.

$$\boxed{\text{No Solution}}$$

★ **Important**: when you solve an equation with variables in the denominator, you **must** check your solutions!

(at least, check that they do not result in 0 denominators when substituted into the original eqn)

Extraneous Solution: (really not a solution at all)

It's a ~~solution~~ value generated by a a correct solution process, that does not check in the original equation.

Ex.:

$$\frac{1}{x-4} = \frac{3}{x+6}$$

Restrictions:

$x \neq 4$

$x \neq -6$

$$\frac{1}{x-4}\left(\frac{x+6}{x+6}\right) = \frac{3}{x+6}\left(\frac{x-4}{x-4}\right)$$

LCD: $(x-4)(x+6)$

$$\frac{x+6}{(x-4)(x+6)} = \frac{3x-12}{(x+6)(x-4)}$$

$$x+6 = 3x-12$$

$$6 = 2x-12$$

$$18 = 2x$$

$$\frac{18}{2} = \frac{2x}{2}$$

$$9 = x$$

$$\boxed{\{9\}}$$

Ex: Solve.

$$\frac{x+2}{x-3} = \frac{x-4}{x-2} \qquad \text{LCD: } (x-3)(x-2)$$

$x \neq 3$
$x \neq 2$

$$\frac{x+2}{x-3}\left(\frac{x-2}{x-2}\right) = \frac{x-4}{x-2}\left(\frac{x-3}{x-3}\right)$$

$$\frac{x^2-2x+2x-4}{(x-3)(x-2)} = \frac{x^2-3x-4x+12}{(x-2)(x-3)}$$

$$x^2-4 = x^2-7x+12$$
$$-x^2 \qquad -x^2$$

$$-4 = -7x+12$$
$$-12 \qquad -12$$

$$-16 = -7x$$

$$\frac{-16}{-7} = \frac{-7x}{-7}$$

$$\frac{16}{7} = x$$

$$\boxed{\left\{\frac{16}{7}\right\}}$$

Does this result in 0 denominators in the original eqn? No.

Ex: Solve.

$$\frac{x}{x+16} = \frac{2}{x-2}$$

Restrictions

$x \neq -16$
$x \neq 2$

LCD: $(x+16)(x-2)$

$$\frac{x}{x+16}\left(\frac{x-2}{x-2}\right) = \frac{2}{x-2}\left(\frac{x+16}{x+16}\right)$$

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Previous example cont'd

$$\frac{x^2-2x}{(x+16)(x-2)} = \frac{2x+32}{(x+16)(x-2)}$$

$$x^2-2x = 2x+32$$
$$\quad -2x-32 \qquad -2x \quad -32$$

$$x^2-4x-32=0$$

$$(x-8)(x+4)=0$$

$x-8=0$	$x+4=0$
$x=8$	$x=-4$

Do these result in 0 denominators when substituted into the original eqn? No.

Solution Set: $\boxed{\{8,-4\}}$

Chapter 7: Solving Linear Systems

7.1 Solving by Graphing (omit this section)

Definitions:

System of equations: group of equations

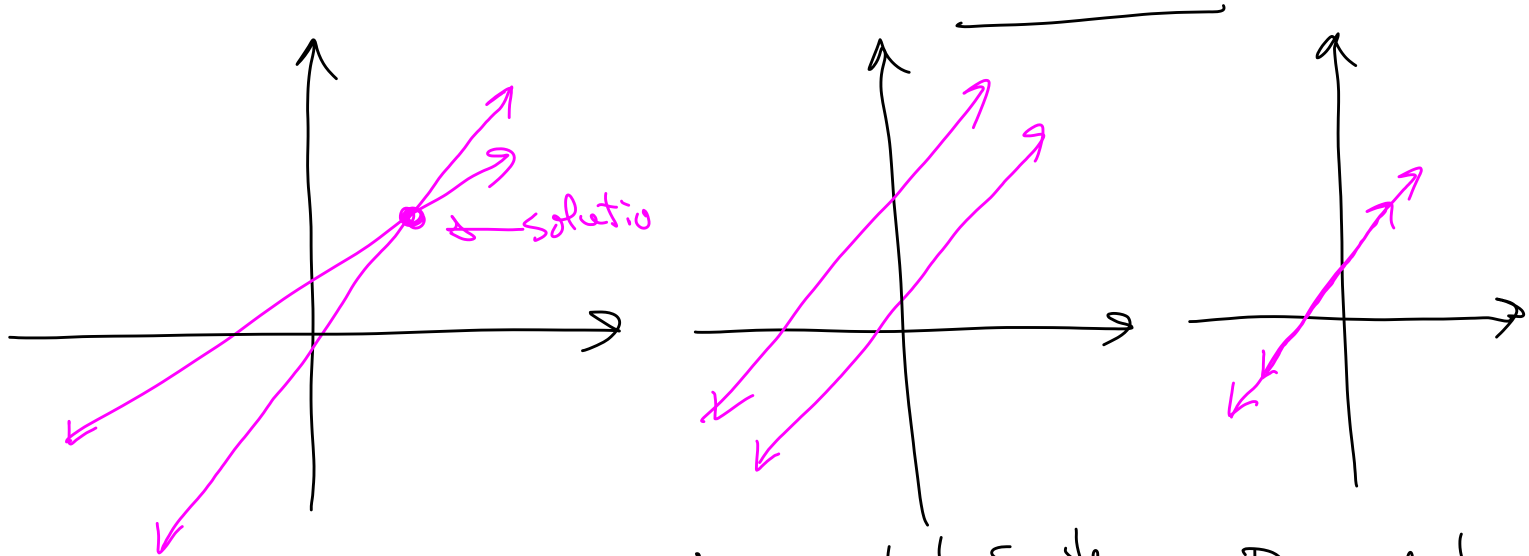
Solution to a system: A set of values for the variables that make all the equations true.

For us: we will solve systems of 2 linear equations in 2 variables (generally be x and y)

Ex: $\begin{cases} 2x - 3y = 4 \\ x + 0y = -12 \end{cases}$ ← graphs of these are lines

A solution is an ordered pair (x, y) that lies on both lines.

3 scenarios for a system of 2 eqns in 2 variables.



Independent System
lines intersect at a single point
(One solution)

Inconsistent System
lines are parallel
(No solution)

Dependent System
lines are the same
(infinitely many solutions)

Test #4 questions

For independent, inconsistent, and dependent systems,

Describe the graph
State the # of solutions
Illustrate with an example graph.

7.2: The substitution method
(for solving linear systems)

Solving by substitution (steps)

1) Solve one equation for one variable (your choice)

2) Substitute this result into the other equation.

3) Solve for the remaining variable.

4) Substitute this value into either equation and solve.

5) Check your answer!

Ex: Solve the system

$$\begin{cases} x-y=9 \\ 2x-11y=-18 \end{cases}$$

1) Solve $x-y=9$ for x.

$$\begin{array}{ccc} x - y & = & 9 \\ +y & & +y \end{array}$$

$$x = 9+y$$

2) Substitute $x=9+y$ into $2x-11y=-18$:

$$2(9+y)-11y=-18$$

3) Solve for y:

$$18+2y-11y=-18$$

$$\begin{array}{ccc} 18-9y & = & -18 \\ -18 & & -18 \end{array}$$

$$-9y=-36$$

$$\frac{-9y}{-9}=\frac{-36}{-9}$$

$$y=4$$

4) Put $y=4$ into $x-y=9$

$$x-4=9$$

$$\begin{array}{ccc} & +4 & +4 \end{array}$$

$$x=13$$

$$\boxed{\begin{array}{c}\underline{\text{Solution:}} \\ (13,4)\end{array}}$$

xyz will ask you to write $x=13, y=4$

5) Check it!

1st eqn

$$x - y = 9$$

$x=13,\ y=4 \Rightarrow 13 - 4 = 9$

$9 = 9$ ✓ True

2nd eqn

$$2x - 11y = -18$$

$$\left.\begin{matrix} x=13 \\ y=4 \end{matrix}\right\} \Rightarrow 2(13) - 11(4) = -18$$

$$26 - 44 = -18$$

$$-18 = -18$$

✓ True

$$\begin{array}{r} 44 \\ -26 \\ \hline 18 \end{array}$$