7.2: Solving Systems by Substitution (cont'd)

Note Title 5/3/2016

Example: Solve the system by substitution.

$$\begin{cases} -4x+3y=19 \\ -6x-5y=0 \end{cases}$$

Step 1: Solve one eqn (your choice) for one variable (your choice).

Here, solve $-6x-5y=0$ for y:

$$\begin{array}{l} -6x \quad\quad -5y=0 \\ +6x \quad\quad +6x \end{array}$$

$$-5y = 6x$$

$$\frac{-5y}{-5} = \frac{6x}{-5}$$

$$y = -\frac{6}{5}x$$

Step 2: Substitute this expression into the other eqn.

$$-4x+3y=19$$

$$-4x+3\left(-\frac{6}{5}x\right)=19$$

Step 3: Solve for the remaining variable.

$$-4x+\frac{3}{1}\left(-\frac{6}{5}x\right)=19$$

$$-4x-\frac{18}{5}x=19$$

$$-\frac{4x}{1}\left(\frac{5}{5}\right)-\frac{18}{5}x=19$$

$$-\frac{20}{5}x-\frac{18}{5}x=19$$

see next page

Previous example cont'd:

$$-\frac{38}{5}x = 19$$

$$\left(-\frac{5}{38}\right)\left(-\frac{38}{5}x\right) = \frac{19}{1}\left(-\frac{5}{38}\right)$$

$$x = -\frac{5}{2}$$

Step 4: Put this value into either equation & solve.

$$-6x - 5y = 0$$

$$x = -\frac{5}{2} \implies -\frac{6}{1}\left(-\frac{5}{2}\right) - 5y = 0$$

$$+\frac{30}{2} - 5y = 0$$

$$15 - 5y = 0$$

$$-15 \qquad -15$$

$$-5y = -15$$

$$\frac{-5y}{-5} = \frac{-15}{-5}$$

$$y = 3$$

Solution: $\boxed{\left(-\frac{5}{2}, 3\right)}$

Step 5: Check your answer (by substituting into both of the original eqns)

$$-6x - 5y = 0$$

$$-\frac{6}{1}\left(-\frac{5}{2}\right) - 5(3) = 0$$

$$\frac{30}{2} - 15 = 0$$

$$15 - 15 = 0 \implies 0 = 0 \checkmark$$

$$-4x + 3y = 19$$

$$-\frac{4}{1}\left(-\frac{5}{2}\right) + 3(3) = 19$$

$$+\frac{20}{2} \quad +9 = 19$$

$$10 + 9 = 19$$

$$19 = 19 \checkmark$$

Ex: Try on your own. Solve by substitution.

$$\begin{cases} 2x - 2y = 2 \\ x - 3y = -7 \end{cases}$$

Solve $2x - 2y = 2$ for x

$$2x = 2 + 2y$$

$$\frac{2x}{2} = \frac{2}{2} + \frac{2y}{2}$$

$$x = 1 + y$$

Put $x = 1 + y$ into $x - 3y = -7$.

$$1 + y - 3y = -7$$

$$1 - 2y = -7$$

$$-2y = -8$$

$$y = \frac{-8}{-2} = 4$$

Put $y = 4$ into $x = 1 + y$ (or into either of my original eqns)

$$x = 1 + y$$

$$y = 4 \Rightarrow x = 1 + 4$$

$$x = 5$$

Soln: $\boxed{(5, 4)}$

Check !!

Ex: $\begin{cases} 5x-8y=7 \\ y=2x-5 \end{cases}$

Solve $y=2x-5$ for y: (already done!)

Substitute $y=2x-5$ into $5x-8y=7$:

$$5x-8(2x-5)=7$$

$$5x-16x+40=7$$

$$-11x=-33$$

$$\frac{-11x}{-11}=\frac{-33}{-11}$$

$$x=3$$

Put $x=3$ into $y=2x-5$:

$$y=2(3)-5$$

$$y=6-5=1$$

Sol'n: $\boxed{(3,1)}$

xyz labs ask you to write $x=3, y=1$

Check it: $5x-8y=7$

$5(3)-8(1)=7$

$15-8=7$

$7=7$ ✓

$y=2x-5$

$1=2(3)-5$

$1=6-5$

$1=1$ ✓

7.3: Solving Systems of Eqns using the Elimination method

Step-by-Step Process

Step 1: Choose a variable to eliminate. Multiply one or both eqns by a strategic number, so that the variable will disappear when we add the eqns.

Step 2: Add the equations

Step 3: Solve for the remaining variable.

Step 4: Either
(a) Plug your value into either eqn & solve.
or (b) Do elimination again to find the other variable

Step 5: Check your solution.

Example: Solve the system by elimination.

$$\begin{cases} 11x - 5y = 2 \\ 3x - y = 1 \end{cases}$$

$11x - 5y = 2 \longrightarrow 11x - 5y = 2$

$3x - y = 1 \xrightarrow{(-5)} -15x + 5y = -5$

Add: $-4x + 0 = -3$

$$-4x = -3$$

$$\frac{-4x}{-4} = \frac{-3}{-4}$$

$$x = \frac{3}{4}$$

You could put $x = \frac{3}{4}$ into either eqn (same as in substitution method). Or, do elimination again, this time getting rid of x:

$11x - 5y = 2 \xrightarrow{(3)} 33x - 15y = 6$

$3x - y = 1 \xrightarrow{(-11)} -33x + 11y = -11$

Add: $0 - 4y = -5$

$$-4y = -5$$

$$\frac{-4y}{-4} = \frac{-5}{-4}$$

$$y = \frac{5}{4}$$

Solution: $\left(\frac{3}{4}, \frac{5}{4}\right)$

(Independent system)
(one solution)

Check it! See next page

Check:

$x = \frac{3}{4}, y = \frac{5}{4}$

$11x - 5y = 2$

$11\left(\frac{3}{4}\right) - 5\left(\frac{5}{4}\right) = 2$

$\frac{33}{4} - \frac{25}{4} = 2$

$\frac{8}{4} = 2$

$2 = 2$ ✓

$3x - y = 1$

$x = \frac{3}{4}$, $y = \frac{5}{4}$

$3\left(\frac{3}{4}\right) - \frac{5}{4} = 1$

$\frac{9}{4} - \frac{5}{4} = 1$

$\frac{4}{4} = 1$

$1 = 1$ ✓

Try on your own: Solve by elimination.

Ex 1) $\begin{cases} -5x + 2y = -6 \\ 10x + 7y = 34 \end{cases}$

Ex 2) $3x - 4y = 7$

$3y + x = 1$

Ex 1:

$-5x + 2y = -6 \xrightarrow{(2)} -10x + 4y = -12$

$10x + 7y = 34 \longrightarrow 10x + 7y = 34$

Add

$11y = 22$

$\frac{11y}{11} = \frac{22}{11}$

$y = 2$

Put $y = 2$ into $-5x + 2y = -6$

$-5x + 2(2) = -6$

$-5x + 4 = -6$

$-5x = -10$

$\frac{-5x}{-5} = \frac{-10}{-5}$

$x = 2$

Solution:

$(2, 2)$

Check it!

Ex 2) $3x - 4y = 7$

$3y + x = 1$ ← Rewrite so the x comes 1st!

$3x - 4y = 7 \longrightarrow 3x - 4y = 7$

$x + 3y = 1 \xrightarrow{(-3)} -3x - 9y = -3$

Add: $-13y = 4$

$\frac{-13y}{-13} = \frac{4}{-13}$

$y = -\frac{4}{13}$

$3x - 4y = 7 \xrightarrow{(3)} 9x - 12y = 21$

$x + 3y = 1 \xrightarrow{(4)} 4x + 12y = 4$

Add: $13x = 25$

$\frac{13x}{13} = \frac{25}{13}$

$x = \frac{25}{13}$

Solution:

$\left(\frac{25}{13}, -\frac{4}{13}\right)$

Check it:

$3x - 4y = 7$

$3\left(\frac{25}{13}\right) - 4\left(\frac{-4}{13}\right) = 7$

$\frac{75}{13} + \frac{16}{13} = 7$

$\frac{91}{13} = 7$

$7 = 7$ ✓ ok

$13 \overline{)91}\ 7$

$x + 3y = 1$

$\frac{25}{13} + 3\left(-\frac{4}{13}\right) = 1$

$\frac{25}{13} - \frac{12}{13} = 1$

$\frac{13}{13} = 1$

$1 = 1$ ✓

Ex: Solve

$$\begin{cases} -x + 6y = 2 \\ 3x - 18y = 5 \end{cases}$$

$-x + 6y = 2 \xrightarrow{(3)} -3x + 18y = 6$

$3x - 18y = 5 \longrightarrow 3x - 18y = 5$

Add: $0 + 0 = 11$

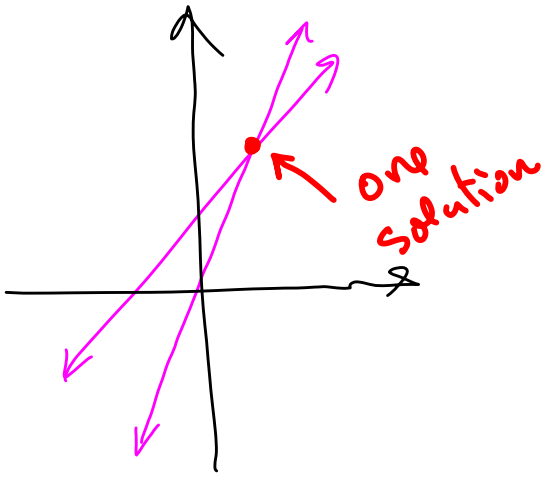
$0 = 11$ False!

No solution

inconsistent system

lines are parallel

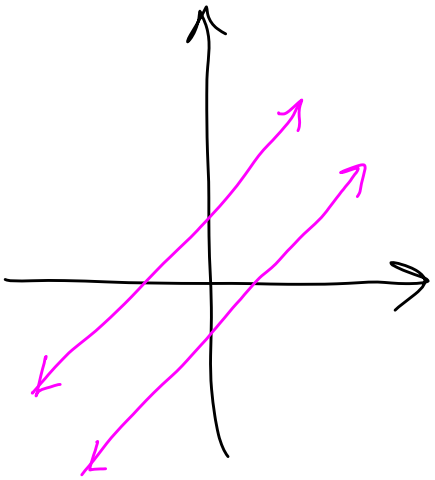
Recall:



Independent system

One solution

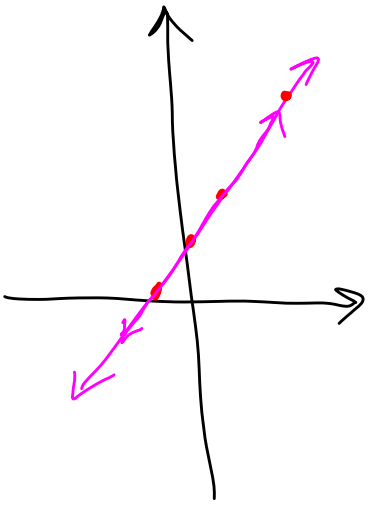
Lines intersect at a single point



Inconsistent system

No solution

lines are parallel



Dependent system

Infinitely many solutions

Both equations represent the same line

Ex:

$$\begin{cases} 2x - 10y = 6 \\ -5x + 25y = -15 \end{cases}$$

$2x - 10y = 6 \xrightarrow{(5)} 10x - 50y = 30$

$-5x + 25y = -15 \xrightarrow{(2)} -10x + 50y = -30$

$0 + 0 = 0$

$0 = 0$ True!

This is not the answer

correct final answer → Dependent system

(infinitely many solutions;

Both eqns represent the same line.)