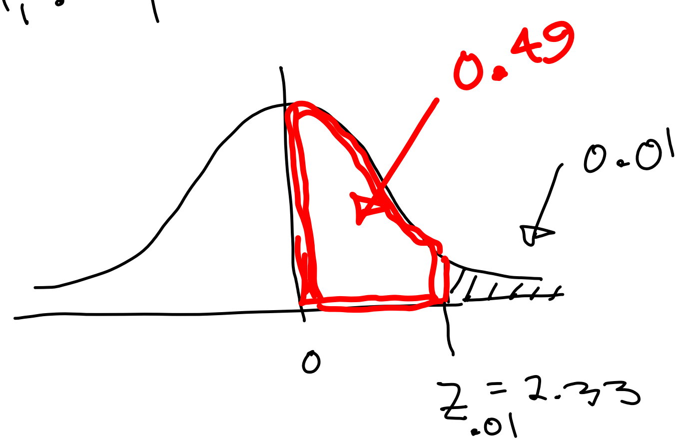
10.2: Hypothesis Testing for a population Proportion (cont'd)

10.2 #16 Manufacturer of a drug claims that more than 94% of patients taking the drug are healed.

In clinical trials, 213 of 224 patients taking the drug were healed. Test the manufacturer's claim at the $\alpha = 0.01$ level of significance.

H_0: $p = 0.94$ (p is proportion of patients who are healed)

H_1: $p > 0.94$ This tells us we are doing a 1-tailed test.



Find critical value of z:

Look up area = 0.49 in z-table. It corresponds to $z = 2.33$

See next page

Calculate z_0 for our sample:

Standard error: $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$

use $p = 0.94$ (value from H_0) $\longrightarrow$ $= \sqrt{\dfrac{0.94(0.06)}{224}}$

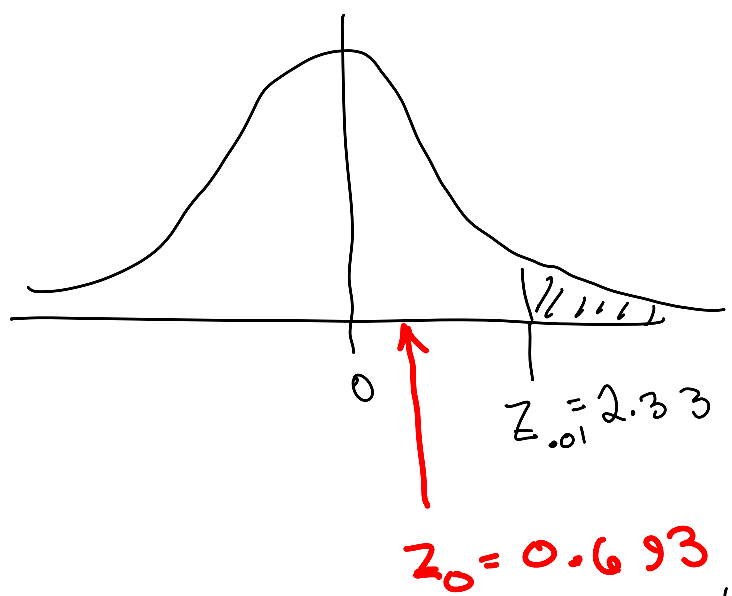
≈ 0.01587

$$z_0 = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{0.951 - 0.94}{0.01587}$$

$= 0.693$

Is this in the rejection region?

(using Sample proportion: $\hat{p} = \dfrac{213}{224} \approx 0.951$)



$z_0 = 0.693$ is not in the rejection region, so we $\boxed{\text{do not reject } H_0.}$

Does this sample provide evidence for the manufacturer's claim that the drug heals more than 94% of people?

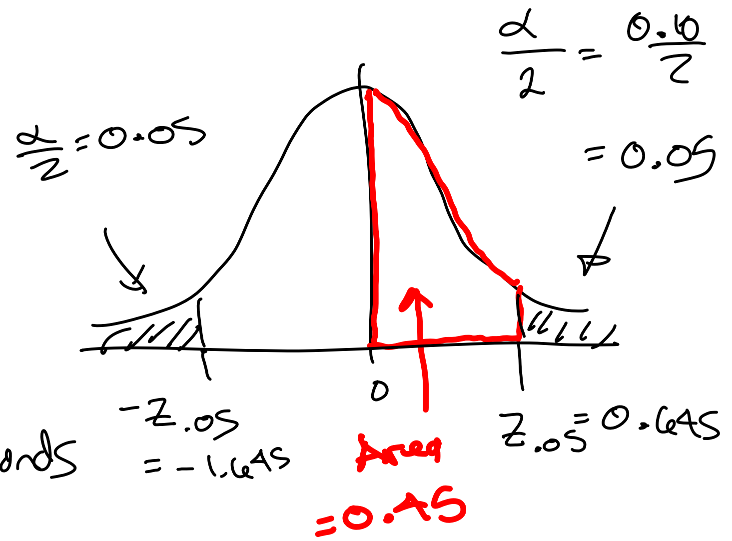
No!

Example: 10.2 #22

In 2000, 58% of females age 15 and older lived alone. A sociologist wants to find out whether this percentage is different today. The sociologist selects a random sample of 500 females (15 and older) and finds that 285 are living alone. Is there sufficient evidence at the $\alpha = 0.10$ significance level to conclude that the proportion of females living alone has changed?

H_0: $p = 0.58$

H_1: $p \neq 0.58$



Find critical value of z:

We look up Area = 0.45 in z-table. It corresponds to $z = 1.645$

Compute test statistic z_0 from sample

$$z_0 = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$$

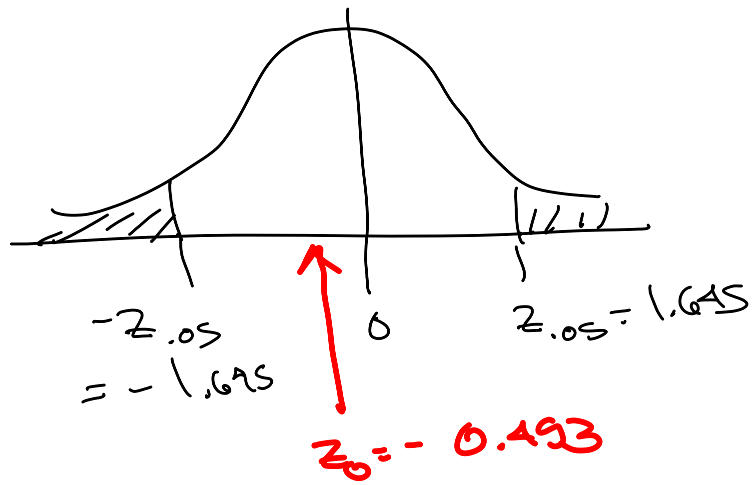
Calculate sample proportion $\hat{p} = \frac{285}{500} = 0.57$

See next page

Calculate standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.58(0.42)}{500}} \approx 0.0220726$$

$$z_0 = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.57 - 0.58}{.0220726} \approx -0.453$$



The test statistic $z_0 = -0.453$ is not in the rejection region, so we

$\boxed{\text{do not reject } H_0.}$

No, this sample does not provide evidence the percentage has changed.

11.1 : Inferences regarding proportions from independent samples

Test statistic: $z_0 = \dfrac{\hat{p}_1 - \hat{p}_2}{\sigma_{\hat{p}_1 - \hat{p}_2}}$

To calculate the standard error, we use a pooled estimate of p:

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

where n_1 and n_2 are the sample sizes, and x_1 and x_2, respectively, are the numbers of data points in the sample that have the characteristic of interest.

The standard error is

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\hat{p}(1-\hat{p})}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Skip the confidence intervals
Skip the dependent samples
Skip finding the sample sizes.

Ex: #26 from 11.1

Clinical trials

standard treatment: 642 of 2105 patients were cured

New treatment: 697 of 2115 patients were cured.

Does the new procedure cure a higher percentage of patients at $\alpha = 0.05$ level?

$x_1 = 642$, $n_1 = 2105$ $\Big\}$ $\hat{p}_1 = \frac{642}{2105} = 0.305$ $\qquad$ $x_2 = 697$, $n_2 = 2115$ $\Big\}$ $\hat{p}_2 = \frac{697}{2115} \approx 0.330$

Pooled estimate of p: $\quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{642 + 697}{2105 + 2115}$

$$= \frac{1339}{4220} \approx 0.317$$

$$1 - \hat{p} = 0.683$$

Standard error:

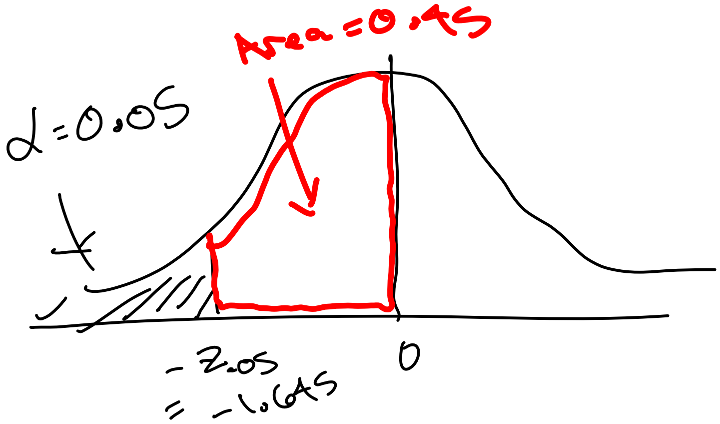
$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$= \sqrt{0.317(0.683)\left(\frac{1}{2105} + \frac{1}{2115}\right)}$$

$$\approx 0.014326$$

Compute test statistic

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sigma_{\hat{p}_1 - \hat{p}_2}} = \frac{0.305 - 0.330}{0.014326} \approx -1.745$$

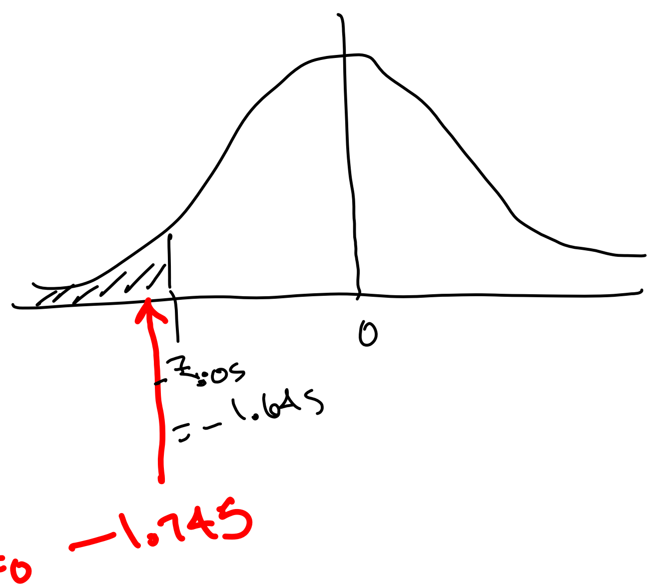


$H_0: p_1 = p_2$

$H_1: p_2 > p_1$

same as $p_1 < p_2$

Find critical value: look up area = 0.45 in table. It corresponds to $z_{.05} = 1.64$



Ours is on the left because $\hat{p}_1 < \hat{p}_2$ (so $\hat{p}_1 - \hat{p}_2$ is negative)

Our $z_0 = -1.745$ falls in the rejection region so we $\boxed{\text{Reject } H_0.}$

This sample provides evidence that the new procedure is effective.

However: note that H_0 would not have been rejected if we had used a 2-tailed test. (see next page)

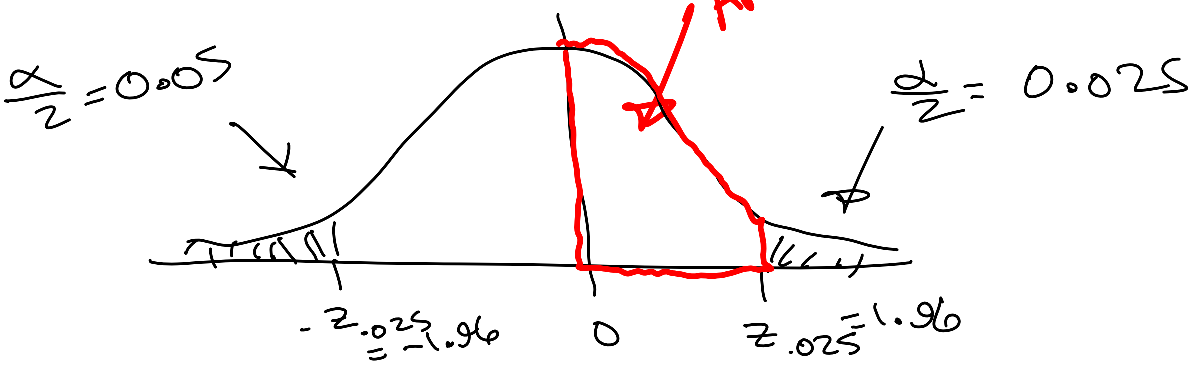
11.1 #26 cont'd

Suppose we used a 2-tailed test:
(the new procedure could be worse)

$H_0: p_1 = p_2$ $\qquad \alpha = 0.05$

$H_1: p_1 \neq p_2$

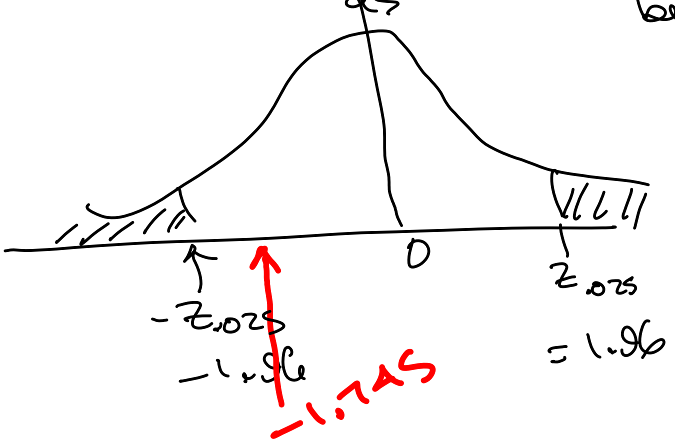
Area $= 1 - 0.025$
$= 0.475$



Find critical value $z_{.05}$: Look up area $= 0.475$ in z-table. It corresponds to a z-score of 1.96

Our computed value for $z_0 = \dfrac{\hat{p}_1 - \hat{p}_2}{\sigma_{\hat{p}-\hat{p}_1}}$

is the same as before $z = -1.745$



Our $z = -1.745$ is no longer in the rejection region, so we do not reject H_0.

See next page

So, this is a example where the Sample provided enough evidence to reject H_0 if we tested it as a one-tail, but not enough to reject it if we test as a 2-tail test.

Researchers need to be very cautious about using a 1-tailed test, because it can appear they may have only done it to get a smaller critical value for z.