10.3: Hypothesis Tests for a Population Mean

How do we decide whether the null hypothesis is tenable? Or whether there is evidence in favor of the alternative hypothesis?

More terminology:

The *p*-value is the probability of obtaining a sample statistic at least as extreme as that observed in the sample, given that the null hypothesis is true.

A result is said to be *statistically significant* if the *p*-value is less than the predetermined α level.

If the *p*-value is less than α, we reject the null hypothesis.
If the *p*-value is greater than or equal to α, we fail to reject the null hypothesis.

Note: We cannot prove the null hypothesis is true. We never accept the null hypothesis. The closest we can come to accepting the null hypothesis is to conclude that there is not enough evidence to reject it.

Procedure for testing a hypothesis about the population mean (critical-value approach):
(classical approach)

This procedure uses the Student *t*-distribution, which assumes a normal population.

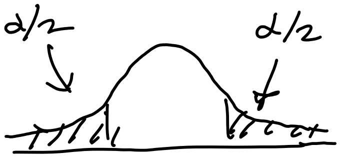
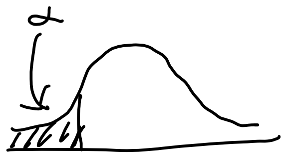
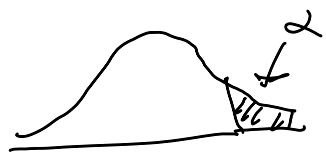
In order to use this procedure, we need to know (or be able to reasonably assume) that the population follows the normal distribution, or have a large sample size ($n \geq 30$).

In addition, the sample should be randomly obtained, observations within the sample must be independent of one another. This means that if we have a sample size that is more than 5% of the population, we should multiply the standard error by a finite population correction factor, $\sqrt{\dfrac{N-n}{n-1}}$. (In this class, I do not anticipate that we will encounter this situation.)

<u>Hypothesis Testing for a Population Mean:</u>

<u>Step 1</u>: Determine the significance level α .

<u>Step 2</u>: Determine the null and alternative hypotheses.

Two-Tailed Test (most common)	Left-Tailed Test (rare)	Right-Tailed Test (rare)
$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu > \mu_0$
 Rejection Region	 Rejection Region	 Rejection Region

Note: One tailed tests assume that the scenario not listed ($\mu > \mu_0$ for a left-tailed test or $\mu < \mu_0$ for a right-tailed test) is not possible or is of zero interest.

<u>Step 3</u>: Use your α level and hypotheses, sketch the rejection region.

<u>Step 4</u>: Compute the test statistic $t = \dfrac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$.

<u>Step 5</u>: Use a table (Table IV, on page A-13) to determine the <u>critical value for *t*</u> associated with your rejection region.

<u>Step 6</u>: Determine whether the value of *t* calculated from your sample (in Step 3) is in the rejection region.

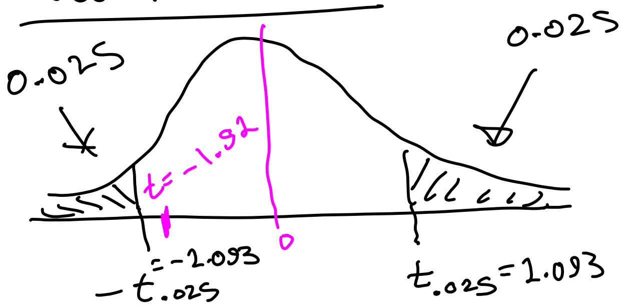
- If *t* is in the rejection region, reject the null hypothesis.
- If *t* is not in the rejection region, do not reject the null hypothesis.

<u>Step 7</u>: State your conclusion.

$\mu = 98.6°F$

Example 1: The normal human body temperature is widely accepted to be 98.6° F and can be assumed to follow a normal distribution. A medical researcher wants to know whether a certain geographical community of Native Alaskans has a mean body temperature of 98.6° F. A sample of 20 members of the Native Alaskan geographical community resulted in a mean body temperature of 98.3° F with a standard deviation of 0.7° F. Perform an appropriate hypothesis test at the 95% confidence level.

Two-tailed test



$H_0: \mu = 98.6°F$

$H_1: \mu \neq 98.6°F$

$\alpha = 1 - 0.95 = 0.05$

Sample

$n = 20$

$\bar{x} = 98.3°F$

$s = 0.7°F$

t-distribution Table:

$df = n - 1 = 19$

right-tail area $= 0.02$

Table $\Rightarrow t_{.025} = 2.093$. This is the critical value of t.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{98.3 - 98.6}{0.7/\sqrt{20}} = -1.92$$

$t = -1.92$ is not in the rejection region, so $\boxed{\text{Do not reject } H_0}$

Example 2: The average amount of lead in the blood of young children is 2 micrograms per deciliter (mcg/dL). A city has recently changed its water supply, and there have been widespread reports of increased lead levels in the water. A concerned doctor wants to dig into the city's medical records to find out whether the children in the city have blood lead levels above the average level of 2 mcg/dL. In a sample of 35 children, she found a mean lead level of 2.60 mcg/dL with a standard deviation of 1.9 mcg/dL. Perform an appropriate hypothesis test at the 95% confidence level.

$H_0: \mu = 2$

$H_1: \mu > 2$

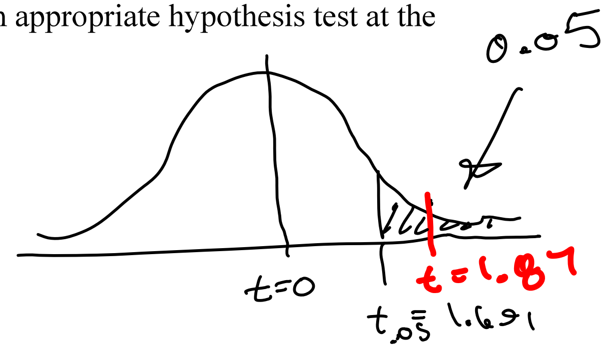
(one-tailed test)

Sample

$n = 35$

$\bar{x} = 2.60$

$s = 1.9$



$\alpha = 0.05$

Use Table

$df = n - 1$

with $df = 34$, right-tail area $= 0.05$.

Table $\Rightarrow$ critical value of t is $t_{.05} = 1.691$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{2.60 - 2}{\frac{1.9}{\sqrt{35}}} = 1.87$$

This t is in the rejection region, so we $\boxed{\text{Reject } H_0.}$

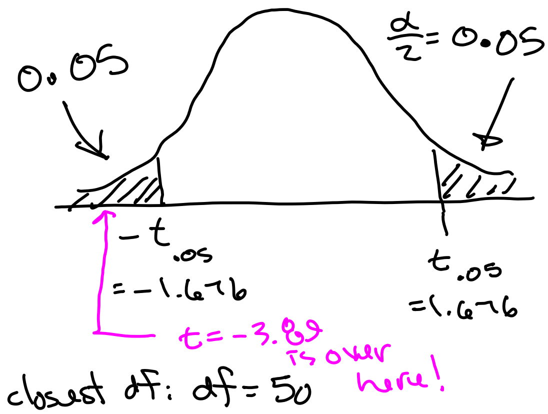
Example 3: Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. A customer (or a FDA analyst) buys 50 bags of chips, weighs them on a high-accuracy scale, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.

$H_0: \mu = 8$ oz

$H_1: \mu \neq 8$ oz

$\alpha = 0.10$

Sample:
$n = 50$
$\bar{x} = 7.89$
$s = 0.2$ oz



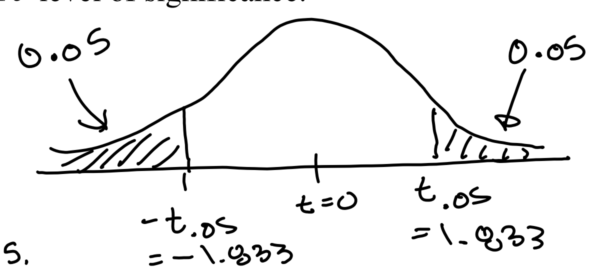
t-distribution table:
We have df = 49, right-tail area = 0.05.
Table doesn't have df = 49, so use the next closest df: df = 50
The critical value of t (from table) is $t_{.05} = 1.676$

Calculate t: $t = \frac{\bar{x} - \mu_{\bar{x}}}{s_{\bar{x}}} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{7.89 - 8}{\frac{0.2}{\sqrt{50}}} = -3.89$. This is in the rejection region, so we $\boxed{\text{Reject } H_0}$

Example 4: Suppose a manufacturer claims on the label that a package contains 8 ounces of potato chips. Again, a customer (or a FDA analyst) wonders whether the package size is accurate. This time, the analysist only buys 10 bags of chips, and obtains a sample mean of 7.89 ounces with a sample standard deviation of 0.2 ounces. Does this sample provide evidence that the manufacturer's labeling may be inaccurate? Use the $\alpha = 0.10$ level of significance.

As before: $H_0: \mu = 8$
$H_1: \mu \neq 8$
$\alpha = 0.10$

Sample:
$n = 10$
$\bar{x} = 7.89$
$s = 0.2$ oz



Use t-table: we have df = 9, right-tail area = 0.05.
From table, the critical value of t is $t_{.05} = 1.833$.

$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{7.89 - 8}{\frac{0.2}{\sqrt{10}}} = -1.74$

This is not in the rejection region, so we $\boxed{\text{Do not reject } H_0.}$