3.1: Measures of Central Tendency

Now, we will begin studying some numerical measures that describe data sets. There are two basic types:

- Measures of central tendency (this section)
- Measures of dispersion (next section)

Summation Notation:

Summation notation is a compact way to write "add up n numbers" of "do something to n numbers first, and then add them up." The numbers are represented as $x_1, x_2, \ldots, x_n$"

Greek Letter Sigma (capital) $\longrightarrow$ $\sum_{i=1}^{n} x_i = x_1 + x_2 + \ldots + x_n$

Stands for "sum"

$x_1 =$ 1st data point
$x_2 =$ 2nd data point
etc
$x_n =$ nth data point (last data point)

Example 1: Consider the numbers 8, 2, 6, 10, 4, 9. Find $\sum_{i=1}^{6} x_i$ and $\sum_{i=1}^{6} x_i^2$.

$\sum_{i=1}^{6} x_i = 8+2+6+10+4+9 = \boxed{39}$

$\sum_{i=1}^{6} x_i^2 = 8^2+2^2+6^2+10^2+4^2+9^2$
$= 64+4+36+100+16+81 = \boxed{301}$

The Mean: Ungrouped Data:

The *mean* of a set of quantitative data is equal to the sum of all the measurements in the data set divided by the total number of measurements in the set.

If $x_1, x_2, \ldots, x_n$ is a set of n measurements, then the *mean*, or *average*, is given by

$$[\text{mean}] = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$ where

Greek letter $\bar{x} =$ [mean] if data set is a sample

mu $\longrightarrow$ $\mu =$ [mean] if data set is the population

We use Greek letters to represent population values (parameters)

We use English letters to represent sample values (statistics)

Find the mean of the data set $\{8, 2, 6, 10, 4, 9\}$. Assume it's a sample.

The mean is $\bar{x} = \frac{8+2+6+10+4+9}{6} = \frac{39}{6} = \boxed{6.5}$

If it were a population, we would write $\mu = \boxed{6.5}$ (mu)

The median:

Sometimes the mean can be misleading for a data set. Suppose that a math class had 7 students with test scores (out of a possible 100) of 88, 99, 7, 78, 89, 94, and 75.

Mean $$\mu = \frac{88+99+7+78+89+94+75}{7} = 75.71$$

The *median* is unaffected by extreme values (outliers). Essentially it is the "middle" of the data set.

Notice: all students except for one got at least 75.

To find the median, you'll need to sort the data in numerical order.

> The Median (Ungrouped Data):
>
> - If the number of measurements is odd, the median is the middle measurement when the measurements are arranged in descending or ascending order.
> - If the number of measurements is even, the median is the mean of the two middle measurements when the measurements are arranged in descending or ascending order.

Example 2: Find the median of the test scores 88, 99, 7, 78, 89, 94, and 75.

7 75 78 88 89 94 99

↑ middle data point is the median

$\boxed{\text{Median} = 88}$

Example 3: Find the median of the test scores 88, 85, 99, 7, 78, 89, 94, and 75.

~~7~~, ~~75~~, ~~78~~, 85, 88, ~~89~~, ~~94~~, ~~99~~

average these to get the median

$$\text{Median} = \frac{85+88}{2} = 86.5$$

Example 4: Provide some everyday examples in which the median is more useful than the mean.

House prices in a neighborhood

Salary

Car prices(?)

Credit scores(?)

The mode:

> The Mode:
>
> The *mode* is the most frequently occurring value in a data set, provided it occurs at least twice. There may be a unique mode, several modes, or no mode.
>
> A data set with two modes is called *bimodal*.

Example 5: Find the median and mode for the following data sets.

a. {4, 5, 5, 5, 5, 5, 6, 7, 8, 12}

median

Mode = 5

Median = 5

b. {1, 2, 3, 3, 3, 5, 7, 7, 7, 23}

Modes: 3 and 7

(bimodal data set)

median = $\frac{3+5}{2} = 4$

Median = 4

c. {1, 3, 5, 6, 7, 9, 11, 15}

No mode

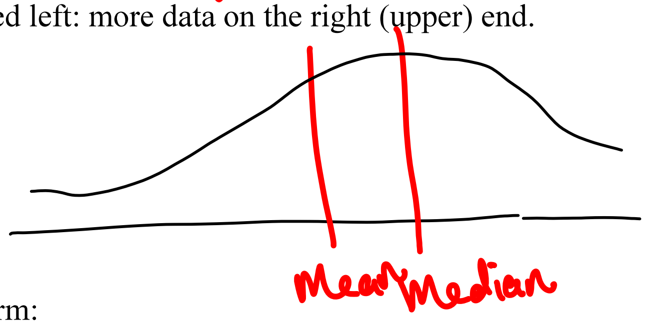
median = 6.5

Example 6:

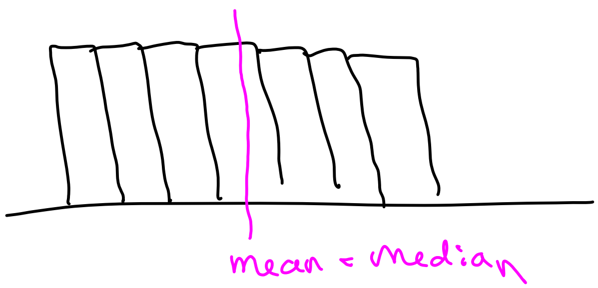
Mean, median, and mode for distributions of different shapes:

Skewed right: more data on the left (lower) end.

Skewed left: more data on the right (upper) end.



Uniform:



Symmetric: