3.5: The Five-Number Summary and Boxplots

The five-number summary:

We can get a fairly useful and descriptive picture of any data set from just 5 numbers: the minimum (smallest value), first quartile, second quartile (median), third quartile, and maximum (largest value).

$Q_2 = m =$ Median

Five-number summary:

Minimum Q_1 Q_2 Q_3 Maximum

Boxplots:

A boxplot, or box-and-whisker plot, visually depicts these five numbers.

How to make a boxplot:

1. Determine the minimum, quartiles, and maximum of the data set.
2. Set up a horizontal scale, and draw a box that has Q_1 and Q_3 for endpoints, and a vertical line at Q_2 (the median). The length of the box is $IQR = Q_3 - Q_1$.
3. Calculate the upper and lower fences, and mark them on the graph:

   Lower fence = $Q_1 - 1.5(IQR)$
   Upper fence = $Q_3 + 1.5(IQR)$

4. Draw a line from Q_1 to the smallest data point that is larger than the lower fence. Draw a line from Q_3 to the largest data point that is smaller than the uper fence.
5. Use an asterisk to mark any data values that lie outside the fences.

5-number summary

min	Q_1	$Q_2 = m$	Q_3	max
3	5	6.5	7.5	11

Example 1: Construct a box plot for the data set.

3, 4, 4, 5, 5, 5, 6, 6, 7, 7, 7, 7, 8, 8, 9, 11

$m = Q_2 = 6.5$

$Q_1 = 5$

$Q_3 = \frac{7+8}{2} = 7.5$

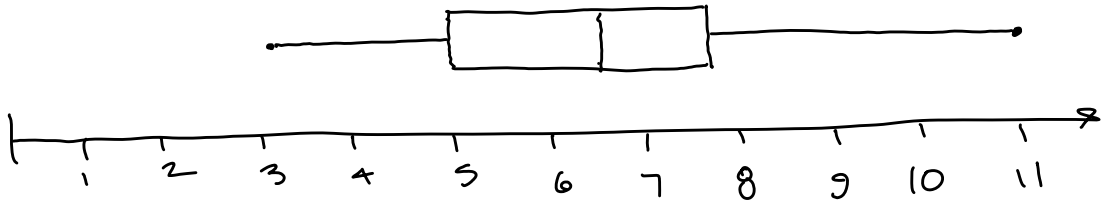
$IQR = 7.5 - 5 = 2.5$

$1.5(IQR) = 1.5(2.5) = 3.75$

Lower fence: $5 - 3.75 = 1.25$

Upper fence: $7.5 + 3.75 = 11.25$

(So no outliers)



Example 2: Construct a box plot for the data set.

25, 1, 5, 3, 7, 14, 12, 10, 5, 9, 12, 4, 6, 13, 2, 8

1, 2, 3, 4, 5, 5, 6, 7, 8, 9, 10, 12, 12, 13, 14, 25

$m = Q_2 = 7.5$

$Q_1 = \frac{4+5}{2} = 4.5$

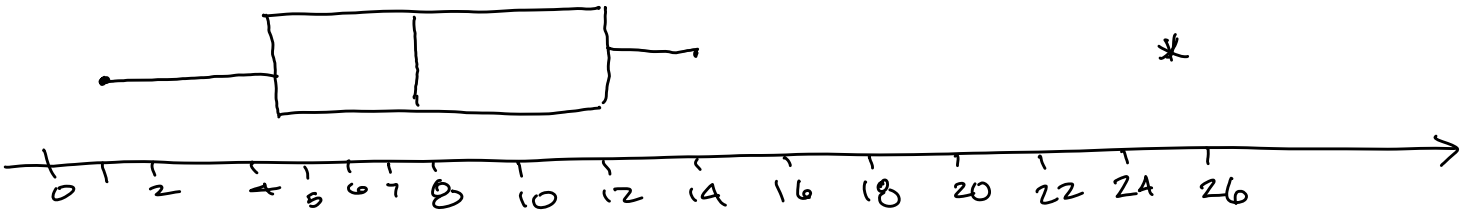
$Q_3 = 12$

$IQR = 12 - 4.5 = 7.5$

$1.5(IQR) = 1.5(7.5) = 11.25$

Lower fence: $4.5 - 11.25 = -6.75$

Upper fence: $12 + 11.25 = 23.25$



5-number summary:

min	Q_1	$Q_2 = m$	Q_3	max
1	4.5	7.5	12	25