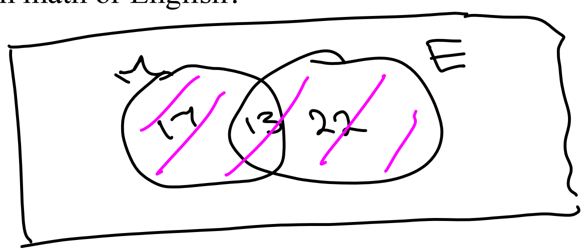
5.2: The Addition Rule and Complements

Example 1: Consider a group of students. 30 of them are enrolled in a math course and 35 are enrolled in an English course. 13 of the students are enrolled in an English course and also a math course. How many students are enrolled in math or English?

M: enrolled in math
E: enrolled in English

$30 - 13 = 17$

$35 - 13 = 22$



Given:

$n(M) = 30$

$n(E) = 35$

$n(E \cap M) = 13$

From Venn diagram,
$n(M \cup E) = 17 + 13 + 22 = \boxed{51}$

these got counted twice

$n(M \cup E) = n(M) + n(E) - n(M \cap E)$

Notation: $n(A)$ means the number of elements in set A.

$= 30 + 35 - 13 = 65 - 13$
$= \boxed{52}$

Addition principle for Counting For any two sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. If A and B are disjoint ($A \cap B = \varnothing$), then $n(A \cup B) = n(A) + n(B)$.

disjoint = mutually exclusive
(don't overlap)

Probability of unions and intersections:

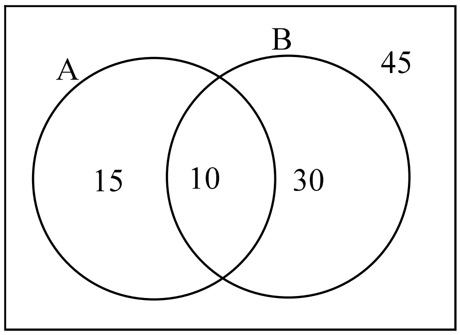
Probability of a Union of Two Events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ If the two events are mutually exclusive (disjoint): $P(A \cup B) = P(A) + P(B)$

This holds even if events are not equally likely

Recall: $A \cup B$ = set of elements that are in A <u>OR</u> B.

$A \cap B$ = set of elements that are in A <u>AND</u> B.

<u>Example 2:</u> Assume that an equally likely sample space is described by the Venn diagram below.



(a) What is $P(A \cup B)$?

(b) what is $P(B)$?

c) What is $P(B^c)$

d) What is $P(B \cap A)$?

(d) $P(B \cap A) = \frac{10}{100} = \boxed{0.10}$

Let's save (a) for last.

(b) $n(S) = 15 + 10 + 30 + 45 = 100$

$P(B) = \frac{n(B)}{n(S)} = \frac{30+10}{100} = \frac{40}{100} = \boxed{0.40}$

(c) $P(B^c) = \frac{15+45}{100} = \frac{60}{100} = \boxed{0.60}$

(a) $P(A \cup B) = \frac{15+10+30}{100}$

$= \frac{55}{100} = \boxed{0.55}$

Note:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{25}{100} + \frac{40}{100} - \frac{10}{100}$

$= \frac{65}{100} - \frac{10}{100} = \frac{55}{100} = \boxed{0.55}$

same

Complements:

Probability of a complement:

$$P(E^C) = 1 - P(E)$$

$$P(E) = 1 - P(E^C)$$

<u>Example 1:</u> Suppose that the probability of someone voting for a certain candidate is 0.46. What is the probability of not voting for the candidate?

E: Person votes for the candidate.

E^c: Person does not vote for the candidate.

$P(E) = 0.46$

$P(E^c) = 1 - 0.46 = 0.54$

Example 2: Consider the data below, from the Congressional Research Service.

https://fas.org/sgp/crs/misc/RS20811.pdf

Table 1. Distribution of Household Money Income by Selected Income Class, 2012

Income Class	# of Households (in thousands)	% of Households
All Households	122,459	100.0
Less than $5,000	4,204	3.4
$5,000 to $9,999	4,729	3.9
$10,000 to $14,999	6,982	5.7
$15,000 to $19,999	7,157	5.8
$20,000 to $24,999	7,131	5.5
$25,000 to $29,999	6,740	5.4
$30,000 to $34,999	6,354	5.2
$35,000 to $39,999	5,832	4.8
$40,000 to $44,999	5,547	4.5
$45,000 to $49,999	5,254	4.4
$50,000 to $59,999	9,358	7.6
$60,000 to $69,999	8,305	6.8
$70,000 to $79,999	7,170	5.9
$80,000 to $89,999	5,969	4.9
$90,000 to $99,999	4,901	4.0
$100,000 to $124,999	9,490	7.7
$125,000 to $149,999	5,759	4.7
$150,000 to $199,999	6,116	5.0
$200,000 to $249,999	2,549	2.1
$250,000 and above	2,911	2.4
Median Income	$51,017	
Mean Income	$71,274	

Source: U.S. Census Bureau, 2012 Annual Social and Economic Supplement to the Current Population Survey.

median is here

↑ 48.6

Probability is 0.077

What is the probability that a randomly selected household has an income of $100,000 or more?

$$P(E) = 0.077 + 0.047 + 0.05 + 0.021 + 0.024 = \boxed{0.219}$$

What is the probability that a randomly selected household has an income below $40,000?

$$P(F) = 0.034 + 0.039 + 0.057 + 0.058 + 0.055 + 0.054 + 0.052 + 0.048 = \boxed{0.397}$$

What is the probability that a randomly selected household has an income below $250,000?

G: Household has income $< \$250K$

G^c: Household has income $\geq \$250K$

$$P(G) = 1 - P(G^c) = 1 - 0.024 = \boxed{0.976}$$

What is the probability that a randomly selected household has an income of \$20,000 or more?

H: Household income is $\geq$ \$20 000

H^c = " " is $<$ \$20 000

$P(H^c) = 0.034 + .039 + 0.057 + 0.058 = 0.188$

Approximate the median household income.

$P(H) = 1 - 0.188 = \boxed{0.812}$

skip this

Odds:

Sometimes the likelihood (or unlikelihood) of an event is described using *odds* instead of probabilities.

Summary:
Probability: The event is contrasted against the whole.
Odds: The event is contrasted against the complement.

Converting from probability to odds:

From Probability to Odds:
• Odds for $E = \dfrac{P(E)}{P(E')}$ • Odds against $E = \dfrac{P(E')}{P(E)}$
When possible, express odds as ratios of whole numbers.

$P(A) = \frac{4}{52} = \boxed{\frac{1}{13}}$

Example 3: What are the odds against ~~rolling~~ drawing an ace when drawing a single card from a standard deck?

4 aces

48 non-aces

odds for getting an ace are $\frac{4}{48} = \frac{1}{12}$ or $\boxed{1:12}$

odds against getting an ace are $\frac{48}{4} = \frac{12}{1} = \boxed{12:1}$

$1 - 0.08 = 0.92$

Example 4: Suppose that, based upon genetics, a child has a 0.08 probability of developing a certain disease. What are the odds against the child developing the disease?

"Chances for the disease" : 8 (out of 100)

"chances against disease" : 92 (out of 100)

odds against getting the disease are $\frac{92}{8} = \frac{23}{2}$ or $\boxed{23:2}$

Converting odds to probability:

From Odds to Probability:

If odds for an event E are $\frac{m}{n}$, (i.e. m:n) then $P(E) = \frac{m}{m+n}$.

complement comes 1st

Example 5: If the odds against a horse winning a race are 7:1, what is the probability that the horse will win?

outcomes/chances for losing: 7

outcomes/chances for winning: 1

Total outcomes/chances: $7+1=8$

So, $P(\text{Win}) = \boxed{\frac{1}{8}}$

Example 6: Suppose an insurance company has used past flood data to determine that determined that the odds against a particular house flooding are 150:1. What is the probability that the house floods?

odds against flood are 150 : 1

chances of no flood

chances of flood

Total chances/outcomes = 151

So, $P(\text{Flood}) = \boxed{\frac{1}{151}} \approx 0.0066$