6.1: Discrete Random Variables

A *random variable* is a quantitative variable that represents the outcomes of a probability experiment.

A *discrete random variable* is a random variable that takes on a finite or countably infinite number of values.

A *continuous random variable* is a random variable that takes on all values on an interval of the real number line (i.e., it is not countable).

A *discrete probability distribution* is a function that assigns a probability to each outcome. (So, it assigns a probability to each value of the discrete random variable). If there are a finite number of outcomes, the sum of all their probabilities must equal 1. Each probability must be between 0 and 1, inclusive. The probability distribution can be described by a table, graph, or mathematical formula.

Mean of a discrete random variable:

The Mean (Expected value) of a Discrete Random Variable**:**

Suppose that a random variable X can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the mean of X is

$E(X) = \mu = x_1 p_1 + x_2 p_2 + ... + x_n p_n$. Also called $E(X)$

Suppose an experiment is repeated many times, and the values of X are recorded and then averaged. As the number of repetitions increases, the average value of X will become closer and closer to μ. For that reason, the mean is called the *expected value* of X. ← $E(X)$

Example 1: A probability distribution is given by the table below. Find the mean (the expected value of X). $P(3) = P(X=3) = 0.15$

x	3	4	5	6	7	8	9
$P(X=x)$	0.15	0.20	0.30	0.12	0.08	0.10	0.05

← Sum is 1

Mean: $\mu = E(X) = 3(0.15) + 4(0.20) + 5(0.3) + 6(0.12)$
$+ 7(0.08) + 8(0.10) + 9(0.05) = \boxed{5.28}$

Example 2: Suppose that an organization sells 1000 raffle tickets for $1 each. One ticket is for a gift basket worth $200, and three tickets are for $50 gift certificates to a restaurant. Find the expected net winnings for a person who buys one ticket.

X = Net winnings for 1 ticket

Outcome	x	$P(X=x)$
Nothing (don't win)	$-\$1$	$\frac{996}{1000} = 0.996$
Gift Basket	$200-1=\$199$	$\frac{1}{1000} = 0.001$
Gift Cert. for restaurant	$50-1=\$49$	$\frac{3}{1000} = 0.003$

Check: do these add up to 1? Yes

Expected Net Winnings

$= E(X) = \mu$

$= -1(0.996) + 0.001(199) + 0.003(49)$

$= \boxed{-0.65}$

Example 3: Suppose the yearly premium for a car insurance policy is $2300 for a customer in a certain category. Statisticians for the insurance company have determined that a person in this category has a 0.007 probability of having an accident that costs the insurance company $100,000 and a 0.015 probability of having an accident that costs the insurance company $30,000. What is the expected value of the insurance policy to the customer? To the insurance company? Let X = net value to customer.

Outcome	x	$P(x)$
Big accident	$100000-2300 = 97700$	0.007
Small accident	$30000-2300 = 27700$	0.015
No accident	-2300	$1-0.007-0.015 = 0.978$

Sum: 1 ✓ ok

Expected value to customer:

$\mu = E(X) = 97700(0.007) + 27700(0.015) - 2300(0.978)$

$= \boxed{-\$1150}$

Expected value to the insurance company is opposite in sign:

$\boxed{\$1150}$

Standard deviation of a discrete random variable:

<u>The Standard Deviation of a Discrete Random Variable</u>**:**

Suppose that a random variable X can take on the n values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the mean of X is

$$\sigma = \sqrt{(x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + ... + (x_n - \mu)^2 p_n}$$

$$= \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p_i}$$

need a square root on this one too!

<u>Example 4:</u> Calculate the mean and standard deviation of the probability distribution. Create a probability histogram.

x	$P(X = x)$ [sometimes written $P(x)$]
0	0.11
1	0.32
2	0.43
3	0.10
4	0.04

<u>Mean:</u>

$\mu = E(X) = 0(0.11) + 1(0.32) + 2(0.43) + 3(0.10) + 4(0.04)$

$= \boxed{1.64}$

<u>Std deviation:</u>

x	$x - \mu$	$(x-\mu)^2$	P	$(x-\mu)^2 P$
0	$0 - 1.64$	$(-1.64)^2 = 2.6896$	0.11	0.295856
1	$1 - 1.64$	$(-0.64)^2 = 0.4096$	0.32	0.131072
2	$2 - 1.64$	$(0.36)^2 = 0.1296$	0.43	0.055728
3	$3 - 1.64$	$(1.36)^2 = 1.8496$	0.10	0.18496
4	$4 - 1.64$	$(2.36)^2 = 5.5696$	0.04	0.222784
				0.8904

variance: $\sigma^2 = 0.8904$

$\sigma = \sqrt{0.8904} \approx \boxed{0.9436}$

Example 5: Use the frequencies to construct a probability distribution for the random variable *X*, which represents the number of games bowled by customers at a bowling alley. Calculate the mean and standard deviation of *X*.

Number of Games	Frequency	Relative Frequency
1	37	37/127
2	45	45/127
3	29	29/127
4	12	12/127
5	4	4/127

$n = 127$

We use the relative frequencies to assign the probabilities in our probability distribution:

x	$P(x)$
1	37/127
2	45/127
3	29/127
4	12/127
5	4/127

(you should check that the probabilities add up to 1)

Mean:

$$\mu = 1\left(\frac{37}{127}\right) + 2\left(\frac{45}{127}\right) + 3\left(\frac{29}{127}\right) + 4\left(\frac{12}{127}\right) + 5\left(\frac{4}{127}\right)$$

$$= \frac{1}{127}\left(1(37) + 2(45) + 3(29) + 4(12) + 5(4)\right) \qquad \left[\text{factor out } \frac{1}{127}\right]$$

$$= \frac{1}{127}(282) = \frac{282}{127} \approx 2.2205$$ ← store in calculator, if possible.

$\boxed{\text{The mean number of games is } 2.2205.}$

x	$x-\mu$	$(x-\mu)^2$	$P(x)$	$(x-\mu)^2 p(x)$
1	1−2.2205	$(-1.2205)^2 \approx 1.48955$	37/127	0.433963
2	2−2.2205	$(-0.2205)^2 \approx 0.04861$	45/127	0.017224
3	3−2.2205	$(0.7795)^2 \approx 0.60766$	29/127	0.138757
4	4−2.2205	$(1.7795)^2 \approx 3.16672$	12/127	0.299218
5	5−2.2205	$(2.7795)^2 \approx 7.72577$	4/127	0.243331

$\sigma^2 = \text{sum} = 1.132493$ ← Variance

$\sigma = \sqrt{\sigma^2}$

$= \sqrt{1.132493}$

$\approx \boxed{1.064}$ std. dev. of # of games