7.2: Applications of the Normal Distribution

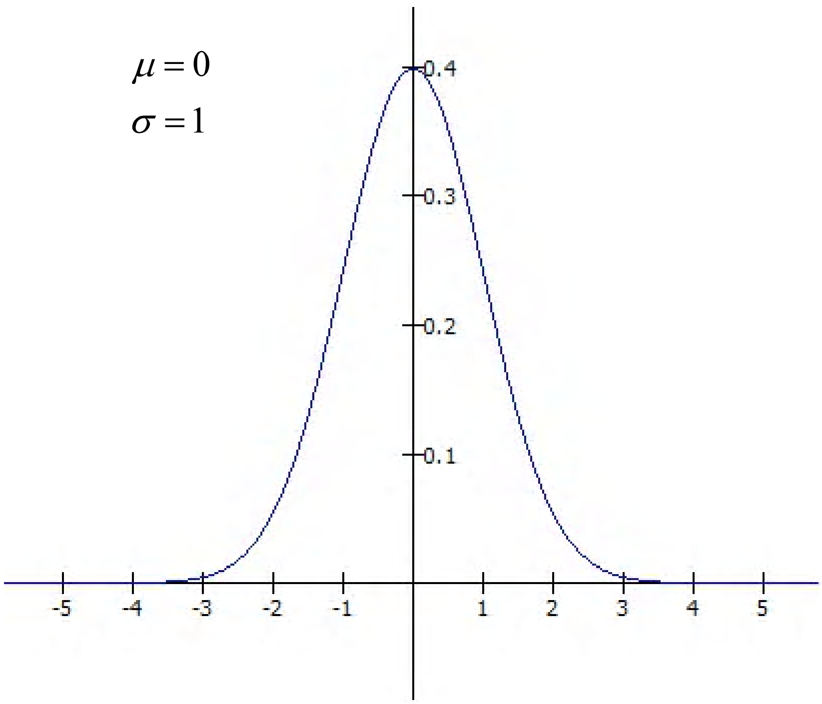
Areas under the normal curve:

One of the many remarkable things about normal curves is the use of area under a normal curve. No matter what the shape of a normal curve, specific areas (and thus probabilities) can be determined easily by using only one table. We *standardize* the values of the variable, by converting them to equivalent values on the *standard normal curve*.

The standard normal curve:

The *standard normal curve* is the normal curve with mean 0 and standard deviation 1. Other normal curves are related to the standard normal curve in this way: the area under any normal curve from μ to $\mu + z\sigma$ corresponds to the area under the standard normal curve from 0 to *z*.

Graph of the standard normal curve:



To standardize the values of a normally distributed variable, we convert them to *z*-scores.

Recall: The *z*-score of a data point is its signed distance from the mean, measured in standard deviations.

Standardizing the values of a normal distribution:

In a normal distribution with mean μ and standard deviation σ, where *x* is a data value, the *z*-score is

$$z = \frac{x-\mu}{\sigma}.$$

The area under a normal curve between $x = a$ and $x = b$ is the same as the area under the standard normal curve between the *z*-score for *a* and the *z*-score for *b*.

Example 1: Consider a normal distribution with mean 10 and standard deviation 3.

a. What is the *z*-score corresponding to $x = 28$?
b. What is the *z*-score corresponding to $x = 7$?
c. What is the *z*-score corresponding to $x = 20$?

Important: The *z*-score is the number of standard deviations between the data point and the mean.

To determine areas under the curve and thus probabilities, we'll use a table. (See Table V in Appendix A, pp. A-11 and A-12.) Probabilities of *x* falling inside a certain range of values are given by the area under the normal curve for that range.

Example 2: Consider a normal curve with mean 7 and standard deviation 2.

a. What is the area under the curve between 7 and 10?

Z-score for $x=10$ is $\dfrac{x-\mu}{\sigma} = \dfrac{10-7}{2}$

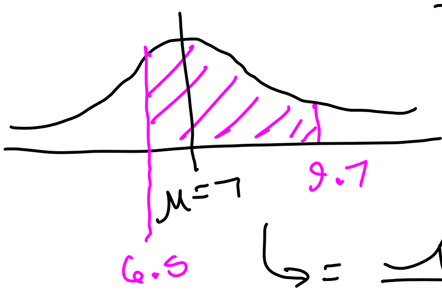
$= \dfrac{3}{2} = 1.50$

From Area handout, Area is $\boxed{0.4332}$

$P(0 < Z < 1.5) = P(7 < X < 10) = \boxed{0.4332}$

b. What is the probability that the variable is between 7 and 10?

c. What is the probability that the variable is between 6.5 and 9.7?

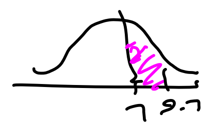


Z-score for 6.5 is $Z_{6.5} = \dfrac{x-\mu}{\sigma} = \dfrac{6.5-7}{2}$

$= -0.25$

Z score for 9.7 is $Z_{9.7} = \dfrac{x-\mu}{\sigma} = \dfrac{9.7-7}{2}$

$= 1.35$

$=$  $+$ 

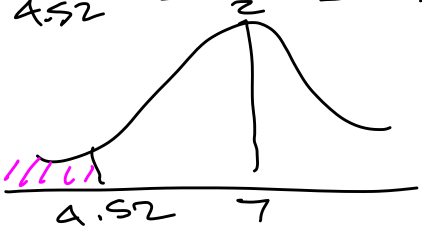
$P(6.5 < X < 9.7)$

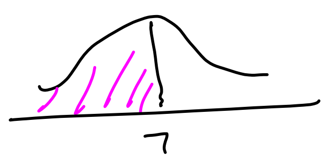
$= 0.0987 + 0.4115$

$= 0.5102$

d. What is the probability that the variable is less than 4.52?

$Z_{4.52} = \dfrac{4.52-7}{2} = -1.24$



$=$  $-$ 

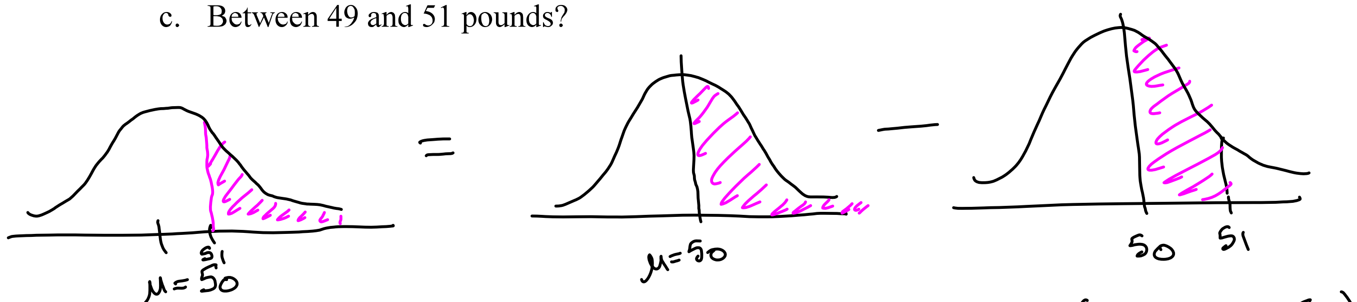
$= 0.5 - 0.3925 = \boxed{0.1075}$

Properties of Normal Probability Distributions:
1. $P(a \le x \le b) =$ area under the curve from a to b.
2. $P(-\infty \le x \le \infty) = 1 =$ total area under the curve.
3. $P(x = c) = 0$.
Note: $P(a \le x \le b) = P(a \le x < b) = P(a < x \le b) = P(a < x < b)$

Example 3: Dusty Dog Food Company ships dog food to its distributors in bags whose weights are normally distributed with a mean weight of 50 pounds and standard deviation 0.5 pound. If a bag of dog food is selected at random from a shipment, what is the probability that it weighs

a. More than 51 pounds?
b. Less than 49 pounds?
c. Between 49 and 51 pounds?

a



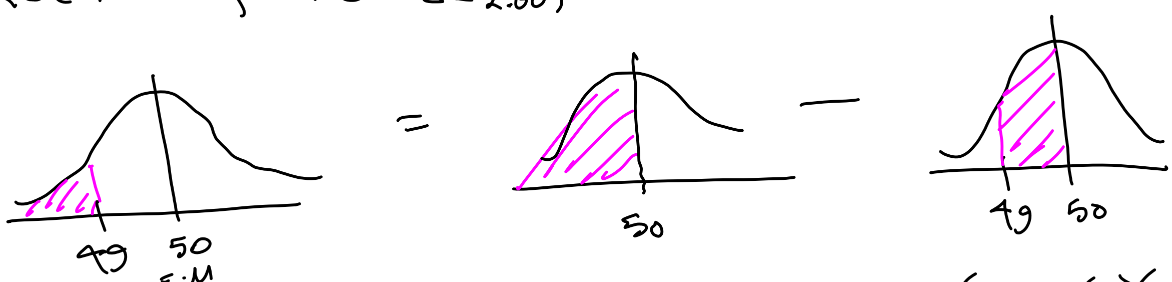
$$P(X > 51) = 0.5 - P(50 < X < 51)$$
$$= 0.5 - P(0 < Z < 2.00)$$
$$= 0.5 - 0.4772 = \boxed{0.0228}$$

Find z-score for 51:

$$z_{51} = \frac{x-\mu}{\sigma} = \frac{51-50}{0.5} = \frac{1}{0.5} = 2.00$$

From Table handout, $P(0 < Z < 2.00) = 0.4772$

b



$$P(X < 49) = 0.5 - P(49 < X < 50)$$
$$= 0.5 - 0.4772$$
$$= \boxed{0.0228}$$

Find z-score for 49:

$$z_{49} = \frac{x-\mu}{\sigma} = \frac{49-50}{0.5} = -2.00$$

From table handout, look up z = 2.00 instead

$P(0 < Z < 2.00 = 0.4772$ (from symmetry)

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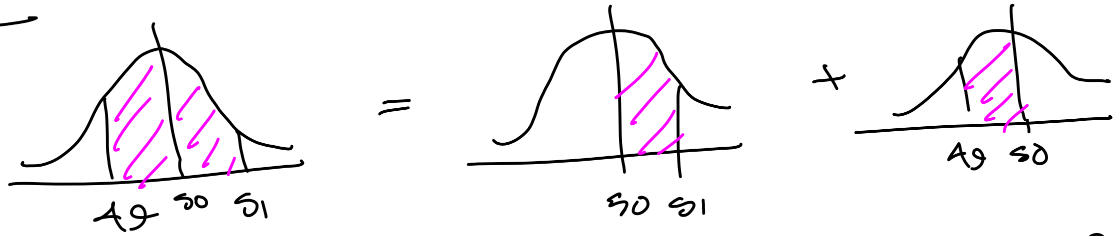
Example 3:

(c)



$$P(49 < X < 51) = 0.4772 + 0.4772$$

$$= \boxed{0.9544}$$

Example 4 Part (c)

$$P(8 < X < 10) = P(7.4 < X < 10) - P(7.4 < X < 8)$$

$$= 0.4850 - 0.1915$$

$$= \boxed{0.2935}$$

From (b)
We know $z_8 = 0.5$
and $P(0 < z < 0.5) = 0.1915$

$$z_{10} = \frac{10 - 7.4}{1.2} = 2.17$$

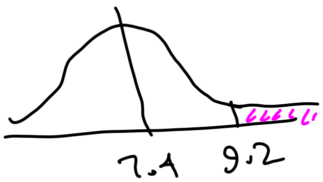
From table,
$P(0 < Z < 2.17) = 0.4850$

Example 4: The medical records of infants delivered at a certain hospital show that the infants' birth weights in pounds are normally distributed with a mean of 7.4 and a standard deviation of 1.2.

$\mu = 7.4, \quad \sigma = 1.2$

a) Find the probability that an infant selected at random from among those delivered at the hospital weighed more than 9.2 pounds at birth.
b) Find the probability that an infant selected at random from among those delivered at the hospital weighed less than 8 pounds at birth.
c) Find the probability that an infant selected at random from among those delivered at the hospital weighed between 8 and 10 pounds.

(a)



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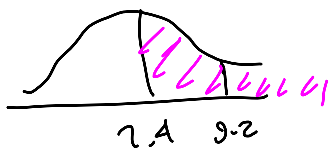

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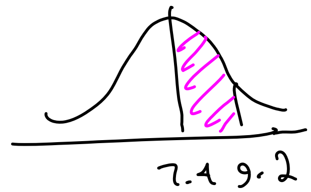


$P(x > 9.2) = 0.5 - P(7.4 < x < 9.2)$

$= 0.5 - 0.4332$

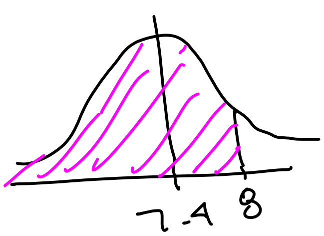
$= \boxed{0.0668}$

$z_{9.2} = \dfrac{9.2 - 7.4}{1.2} = 1.50$

From Table,

$P(0 < z < 1.5) = 0.4332$

(b)



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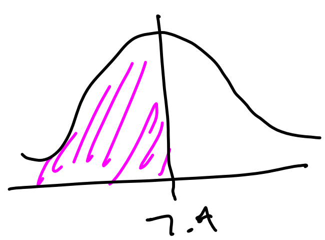

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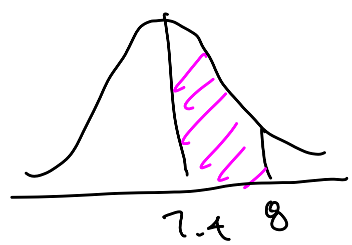

$P(x < 8) = 0.5 + P(7.4 < x < 8)$

$= 0.5 + 0.1915$

$= \boxed{0.6915}$

$z_8 = \dfrac{8 - 7.4}{1.2} = 0.50$

Area $= P(0 < z < 0.5)$

$= 0.1915$

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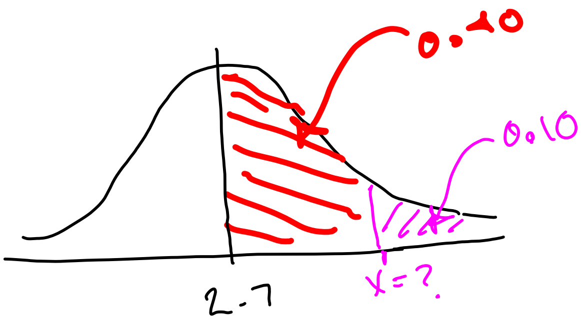
Example 5: The GPA of the senior class of a certain high school is normally distributed with a mean of 2.7 and a standard deviation of 0.4 point. If a senior in the top 10% of his or her class is eligible for admission to any state university, what is the minimum GPA that a senior should have to ensure eligibility to a state university?



$\mu = 2.7$

$\sigma = 0.4$

Look in Table Handout and find closest area to 0.40.

From table, choose Area = 0.4015.

This corresponds to $z = 1.29$

To make the cutoff, the student must be 1.29 std devs above the mean:

$z = \frac{x-\mu}{\sigma}$ OR $x = 2.7 + 1.29(0.40)$

$1.29 = \frac{x-2.7}{0.40}$ $x = 3.216$

$1.29(0.4) = x - 2.7$

$1.29(0.4) + 2.7 = x$

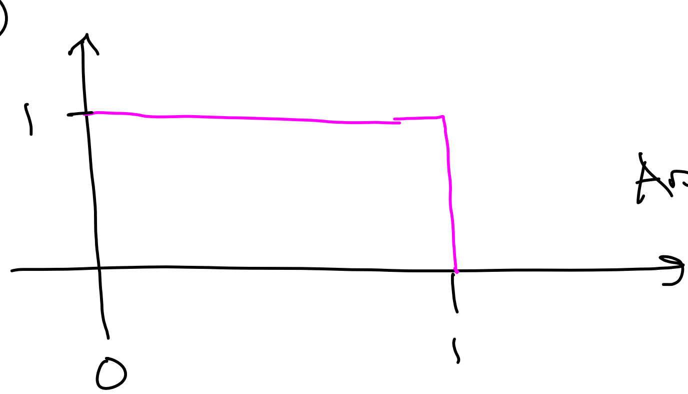
$x = 3.216$

The required GPA is about 3.216

HW Qs over 7.1

7.1 #17 X follows a uniform distribution on the interval $[0,1]$.

(a)



width $= 1-0 = 1$

height $= ?$

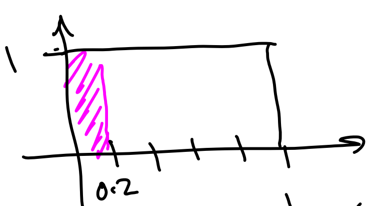
Area $=$ (width) (height) $= 1$

Put in width $=1$:

1 (height) $= 1$

height $= 1$

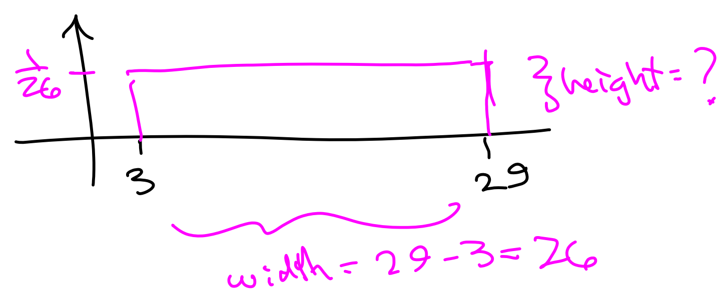
(b) Find $P(0 < X < 0.2)$



$P(0 < x < 0.2) =$ (width) (height)

$= 0.2(1) = \boxed{0.2}$

<u>Note:</u> If X were uniformly distributed from 3 to 29, then:



width(height) $= 1$

26 (height) $= 1$ $\Rightarrow$ height $= \frac{1}{26}$