Supplement: Basic Set Theory

Definition: A *set* is a well-defined collection of objects. Each object in a set is called an *element* of that set.

Examples of sets:

Set of NBA Players in Jan 2016

set of integers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Not sets:

collection of tall people

collection of cute dogs

Sets can be finite or infinite.

Examples of finite sets:

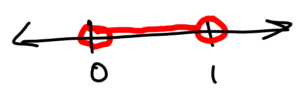
$\{1, 2, 3, 4, 5\}$

{math 1314, math 1324, math 1342}

Examples of infinite sets:

Integers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Any interval on the real number line: $(0,1)$



Notation:

- We usually use capital letters for sets.
  We usually use lower-case letters for elements of a set.

$\in$: Greek letter epsilon

- $a \in A$ means a is an element of the set A.
  $a \notin A$ means a is not an element of the set A.

$a \in A$

$a \notin A$

- The *empty set* is the set with no elements. It is denoted $\varnothing$. This is sometimes called the *null set*.

$\varnothing$

- $S = \{x \mid P(x)\}$ means "S is the set of all x such that $P(x)$ is true". (called rule notation or set roster notation).

  Example: $S = \{x \mid x \text{ is an even positive integer}\}$ means $S = \{2, 4, 6, 8, \ldots\}$

  ← such that

- $n(A)$ means the number of elements in set A.

For $A = \{\text{Black, Blue, Green}\}$. $n(A) = 3$

Definition: We say two sets are *equal* if they have exactly the same elements.

(Note: It is still the same set if the order is rearranged)

Subsets:

Definition: If each element of a set A is also an element of set B, we say that A is a *subset* of B. This is denoted $A \subseteq B$ or $A \subset B$. If A is not a subset of B, we write $A \not\subset B$. $A \subseteq B$ or $A \subset B$

Definition: We say A is a *proper subset* of B if $A \subseteq B$ but $A \neq B$. (In other words, every element of A is also an element of B, but B contains at least one element that is not in A.)

Note on notation: Some books use the symbol $\subset$ to indicate a proper subset. Some books use $\subset$ to indicate any subset, proper or not.

or universe

Definition: The set of all elements under consideration is called the *universal set*, usually denoted U.

Example: If you're dealing with sets of real numbers, then U is the set of all real numbers. So "Wednesday" would not be an element of U, but 5.7 would be in U.

Example 1: Consider these sets.

$B \not\subseteq A$

$$A = \{1,2,3,4,5,6\}$$
$$B = \{1,2,3,4,5,6,7,8\}$$
$$C = \{1,3,5,2,4,6\}$$

$A \subseteq B$

$C \subseteq B$

$A = C$

Also

$A \subseteq A$

$C \subseteq A$

$C \subseteq C$

Note:

- $\varnothing$ is a subset of every set. (i.e. $\varnothing \subseteq A$ for every set A.)
- Every set is a subset of itself. (i.e. $A \subseteq A$ for every set A.)

Example 2: List all subsets of $\{1,2,3\}$.

$\{1\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{2\}$, $\{3\}$, $\{1,2,3\}$, $\varnothing$

Note: If a set has n elements, how many subsets does it have?

It can be proven that a set of n elements has 2^n subsets

So a set of 3 elements has $2^3 = 8$ subsets

A set of 4 elements would have $2^4 = 16$ subsets

Set operations:

- Union $\cup$: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
  (could be in both)
- Intersection $\cap$: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Complement A' or A^C or $A^\sim$: $A' = \{x \in U \mid x \notin A\}$.

A' or A^c or $A^\sim$

our book

Note: $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.

$(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$.

Definition: We say that A and B are *disjoint sets* if $A \cap B = \varnothing$.

(in other words, they share no common elements; they don't overlap)

universe

Example 3: $U = \{1,2,3,4,5,6,7,8\}$

$H = \{1,3,5,7\}$

$K = \{1,2,3\}$

$J = \{2,4,6,8\}$

$L = \{1,2\}$

$J \cap H = \emptyset$ so J and H are disjoint

$H \cap K = \{1,3\}$

$H \cup L = \{1,2,3,5,7\}$

$L^c = \{3,4,5,6,7,8\}$

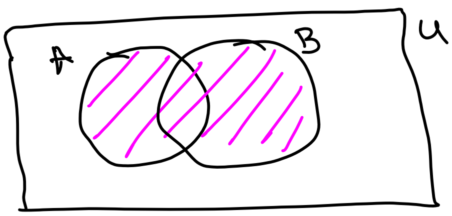
$K^c = \{4,5,6,7,8\}$

$H^c = \{2,4,6,8\}$

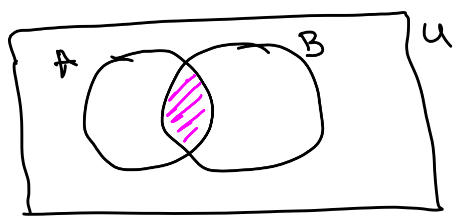
$H^c = J$, $J^c = H$

Venn Diagrams: These help us visualize set relationships and operations.

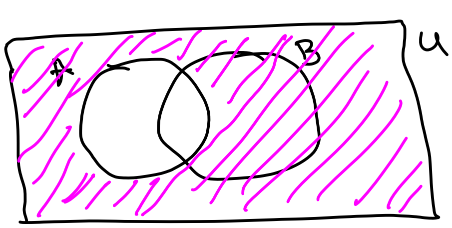
Example 4: Draw Venn diagrams for $A \cup B$, $A \cap B$, A^C, B^C, $(A \cap B)^C$, and $(A \cup B)^C$.



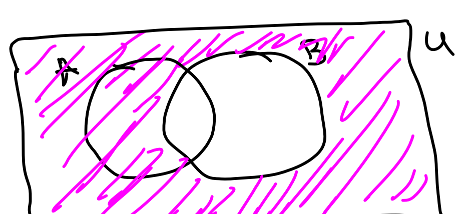
$A \cup B$



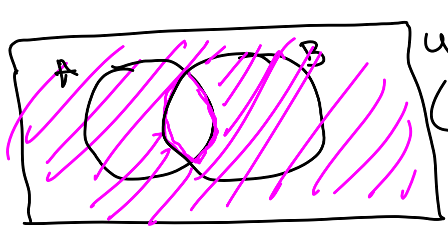
$A \cap B$



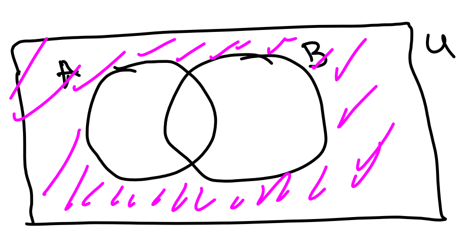
A^c



B^c



$(A \cap B)^c$



$(A \cup B)^c$