

Definition of Linear Equation In One Variable

An **equation** is a statement that indicates two quantities are equal. (has an equal sign).

An **expression** does not have an equals sign.

A **solution to an equation** is a value for the variable that makes the equation a true statement.

To **solve** an equation means to find the solution(s).

Equation

Solution

Check

$$2y + 5 = 11$$

3

$$\text{Substitute 3 for } y: 2(3) + 5 \stackrel{?}{=} 11$$

$$6 + 5 = 11 \checkmark$$

Determine whether the given number is a solution to the equation.

$$1. \quad 3x - 5 = 10; -5$$

$$3(-5) - 5 = 10$$

$$-15 - 5 = 10$$

$$-20 = 10 \text{ False!}$$

So -5 is not a solution

$$2. \quad 3x - 5 = 7; 4$$

$$3(4) - 5 = 7$$

$$12 - 5 = 7$$

$$7 = 7 \text{ True.}$$

Expressions get simplified
Equations get solved.

Yes, 4 is a solution.

Definition: Linear Equation in One Variable

Let a , b , and c be real numbers such that $a \neq 0$.

A **linear equation in one variable** is an equation that can be written in the form $ax + b = c$.

Note: A linear equation in one variable is also called a first-degree equation because the variable x is to the 1st power.

Addition and Subtraction Properties of Equality

Addition and Subtraction Properties of Equality Let a , b , and c represent algebraic expressions.

1. **Addition property** of equality: If $a = b$
then $a + c = b + c$

2. **Subtraction property** of equality: If $a = b$
then $a - c = b - c$

Solve the equations.

Goal: isolate the variable.

$$3. \quad x - 7 = 15$$

$$+7 \quad +7$$

$$x - 7 + 7 = 15 + 7$$

$$x + 0 = 15 + 7$$

$$x = 15 + 7$$

$$x = 22$$

Sol'n Set:

$\{22\}$

The solution is 22.

$$\text{Check: } 22 - 7 = 15$$

$$15 = 15 \checkmark$$

$$4. \quad x + \frac{1}{8} = -\frac{3}{4}$$

$$x = -\frac{3}{4} - \frac{1}{8}$$

$$x = -\frac{3}{4} \left(\frac{2}{2}\right) - \frac{1}{8}$$

$$x = -\frac{6}{8} - \frac{1}{8}$$

$$x = -\frac{7}{8}$$

$\{-\frac{7}{8}\}$

$$5. \quad 1.7 = x + 2.8$$

$$-2.8 \quad -2.8$$

$$1.7 - 2.8 = x$$

$$-1.1 = x$$

Solution Set: $\{-1.1\}$

$$\begin{array}{r} 2.8 \\ -1.7 \\ \hline 1.1 \end{array}$$

Check: $x = -3 \Rightarrow 17 - 19 = 4(-3) - 8 - 3(-3) + 9$
 $-2 = -12 - 8 + 9 + 9$
 $-2 = -20 + 18$
 $-2 = -2 \checkmark \text{OK}$

Simplify first, then solve the equation.

6. $-7 - 43x + 44x = 19 - 3$

$$\begin{array}{r} -7 + x = 16 \\ +7 \quad +7 \end{array}$$

$$x = 16 + 7$$

$$x = 23$$

$$\boxed{\text{Sol'n Set: } \{23\}}$$

7. $17 - 19 = 4x - 8 - 3x + 9$

$$\begin{array}{r} -2 = x + 1 \\ -1 \quad -1 \end{array}$$

$$-3 = x$$

$$\boxed{\{-3\}}$$

Multiplication and Division Properties of Equality

Multiplication and Division Properties of Equality

Let a , b , and c represent algebraic expressions, $c \neq 0$.

1. **Multiplication property** of equality:

If $a = b$,
 then $a \cdot c = b \cdot c$

2. **Division property** of equality:

If $a = b$,
 then $\frac{a}{c} = \frac{b}{c}$

This means we can multiply or divide both sides of an equation by the same non zero number.

Solve the equation by using the division property of equality.

8. $9x = 45$

$$\frac{9x}{9} = \frac{45}{9}$$

$$x = 5$$

$$\boxed{\text{Sol'n Set: } \{5\}}$$

9. $-7m = 98$

$$\frac{-7m}{-7} = \frac{98}{-7}$$

$$m = -\frac{98}{7}$$

$$m = -14$$

$$\boxed{\text{Sol'n Set: } \{-14\}}$$

10. $34.5 = 1.5x$

$$\frac{34.5}{1.5} = \frac{1.5x}{1.5}$$

$$\frac{34.5}{1.5} = x$$

$$x = \frac{34.5}{1.5} \left(\frac{10}{10} \right)$$

$$x = \frac{345}{15} = 23$$

$$\boxed{\{23\}}$$

$$\begin{array}{r} 23 \\ 15 \overline{) 345} \\ \underline{30} \\ 45 \end{array}$$

Solve the equation by using the multiplication property of equality.

11. $\frac{y}{4} = 16$

$$\left(\frac{4}{4} \right) \frac{y}{4} = \frac{16}{1} \left(\frac{4}{4} \right)$$

$$y = 64$$

$$\boxed{\{64\}}$$

12. $-\frac{z}{3} = 120$

$$13. \frac{2}{5}x = 150$$

$$\left(\frac{5}{2}\right) \frac{2}{5}x = \frac{150}{1} \left(\frac{5}{2}\right)$$

$$x = \frac{375}{1}$$

$$x = 375$$

$$\boxed{\{375\}}$$

$$\begin{array}{r} 275 \\ 5 \\ \hline 375 \end{array}$$

$$14. -\frac{3}{4}w = -\frac{9}{5}$$

$$\left(-\frac{4}{3}\right) \left(-\frac{3}{4}w\right) = -\frac{9}{5} \left(-\frac{4}{3}\right)$$

$$w = \frac{36}{15} = \frac{12}{5}$$

$$\boxed{\left\{\frac{12}{5}\right\}}$$

Translations

Write an algebraic equation to represent each English sentence. Then solve the equation.

15. The sum of negative ten and a number is twenty.

16. The quotient of a number and negative ten is twenty.

17. The difference of a number and negative ten is twenty.

18. The product of negative ten and a number is twenty.