Linear Equations Involving Multiple Steps

To understand the proper steps in solving a linear equation, always remember that the ultimate goal is to isolate the variable.

Solve the equations.

1. $7z+5=-9$

$-5 \quad -5$

$7z = -9-5$

$7z = -14$

$\frac{7z}{7} = \frac{-14}{7}$

$z = -2$

$\{-2\}$

2. $-3b-\frac{2}{3}=4$

$+\frac{2}{3} \quad +\frac{2}{3}$

$-3b = 4+\frac{2}{3}$

$-3b = \frac{4}{1}\cdot\frac{3}{3}+\frac{2}{3}$

$-3b = \frac{12}{3}+\frac{2}{3}$

$-3b = \frac{14}{3}$

$\frac{-3b}{-3} = \frac{\frac{14}{3}}{-3}$

$\frac{1}{-3}(-3b) = \frac{14}{3}\left(\frac{1}{-3}\right)$

$b = \frac{14}{-9}$

$b = -\frac{14}{9}$

$\left\{-\frac{14}{9}\right\}$

Dividing by a number is the same as multiplying by its reciprocal.

Note: all equal

$\frac{2}{-8} = -\frac{1}{4}$

$\frac{2}{-8} = -\frac{1}{4}$

$-\frac{2}{8} = -\frac{1}{4}$

3. $-2v+5=3v-20$

$-5 \quad -5$

$-2v = 3v-25$

$-3v \quad -3v$

$-5v = -25$

$\frac{-5v}{-5} = \frac{-25}{-5}$

$v = 5$

Sol'n Set: $\{5\}$

(not done during class)

4. $0.3y+8=0.5y+20$

$0.3y-0.5y = 20-8$

$-0.2y = 12$

$\frac{-0.2y}{-0.2} = \frac{12}{-0.2}$

$y = \frac{12}{-0.2}\left(\frac{10}{10}\right)$

$y = \frac{120}{-2}$

$y = -60$

(not done during class)

Sol'n Set: $\{-60\}$

Procedure for Solving a Linear Equation in One Variable

To solve an equation requiring multiple steps:

1. Simplify both sides of the equation.
2. Apply the addition or subtraction property of equality to collect the variable terms on one side of the equation.
3. Apply the addition or subtraction property of equality to collect the constant terms on the other side of the equation.
4. Use the multiplication or division property of equality to obtain a coefficient of 1 on the variable.

5. $6-(2x-3)=-5x+18$

$6-1(2x-3)=-5x+18$

$6-2x+3=-5x+18$

$9-2x=-5x+18$

$-9 \quad -9$

$-2x=-5x+18-9$

$-2x=-5x+9$

$+5x \quad +5x$

$-2x+5x=9$

$3x=9$

$\frac{3x}{3}=\frac{9}{3}$

$x=3$

$\{3\}$

6. $-5-2(y-5)=4y-2(y+4)-3$

$-5-2y+10=4y-2y-8-3$

$-2y+5=2y-11$

$+2y \quad +2y$

$5=4y-11$

$+11 \quad +11$

$16=4y$

$\frac{16}{4}=\frac{4y}{4}$

$4=y$

soln set: $\{4\}$

Equivalent Equations are equations that have the same solution set.

Conditional Equations, Identities, and Contradictions

I. A conditional equation is an equation that is true for some values of the variable but false for other values.

$x+3=5$

$-3 \quad -3$

$x=2$ $\Rightarrow$ The solution set is $\{2\}$.

The value 2 makes the eqn true; All other values make it false.

$x+3=5$ and $x=2$ are equivalent equations.

II. A contradiction is an equation that has no solution.

$x+3=x+1$

$-3 \quad -3$

$x=x-2$

$-x \quad -x$

$x-x=-2$

$0=-2$ False for every x.

So $x+3=x+1$ has no solution.

Solution Set: $\emptyset$ (symbol for the "empty set")

Therefore, because all steps are equivalent, all steps (including the original eqn) must be false for every x.

III. An identity is an equation that has all real numbers as its solution.

Identity: An equation that is true for all values of the variable.

$x+7=x+7$ ← true for every x

$x+7=x+7$

$-7 \quad -7$

$x=x$

$-x \quad -x$

$0=0$ ← true for every x

Solution Set: all real numbers

Identify the equation as a conditional equation, a contradiction, or an identity. Then describe the solution.

7. $2(h+6)-7=4h-2(h+7)$

$2h+12-7=4h-2h-14$

$2h+5=2h-14$

$-5 \quad -5$

$2h=2h-19$

$-2h \quad -2h$

$0=-19$ false for every h

No Solution

(contradiction)

8. $-2.3p+6=6-2.3p$

$-6 \quad -6$

$-2.3p=-2.3p$

$\frac{-2.3p}{-2.3}=\frac{-2.3p}{-2.3}$ true for every p

$p=p$ always true

Sol'n Set: All real numbers

This is an identity

9. $\frac{2}{3}g+5=\frac{1}{3}g-4$

$\frac{2}{3}g-\frac{1}{3}g=-4-5$

$\frac{1}{3}g=-9$

$\left(\frac{3}{1}\right)\frac{1}{3}g=-\frac{9}{1}\left(\frac{3}{1}\right)$

$g=-27$

sol'n set: $\{-27\}$

This is a conditional equation.

10. $2x-(5x+6)=3(x-5)+9$

$2x-5x-6=3x-15+9$

$-3x-6=3x-6$

$-3x \quad -3x$

$-6x-6=-6$

$+6 \quad +6$

$-6x=0$

$\frac{-6x}{-6}=\frac{0}{-6}$

$x=0$

Sol'n set: $\{0\}$

Note: Two numbers are reciprocals if their product is 1.

Ex: $-\frac{4}{5}$ and $-\frac{5}{4}$ are reciprocals. Ex: $\frac{1}{4}$ and 4 are reciprocals.

11. If an equation has no solution, then is the equation a conditional equation, an identity, or a contradiction?

An equation with no solution is considered a contradiction.

Check of #10:

$2x-(5x+6)=3(x-5)+9$

$x=0 \Rightarrow 2(0)-(5(0)+6)=3(0-5)+9$

$0-(0+6)=3(-5)+9$

$0-6=-15+9$

$-6=-6$ ✓ True!