Recall: $<$ means "is less than" $\leq$ "is less than or equal to"
$>$ means "is greater than" $\geq$ "is greater than or equal to"

Graphing Linear Inequalities

Definition: Linear Inequality in One Variable

A linear inequality in one variable, x, is defined as any relationship of the form:
$ax+b<c$, $ax+b\leq c$, $ax+b>c$, or $ax+b\geq c$, where $a\neq 0$.

Examples of linear inequalities in one variable

$2x+5<8$ $\qquad -6c+1>0$ $\qquad v\geq -9$ $\qquad 6.2y\geq 9$

Definition: Compound Inequality

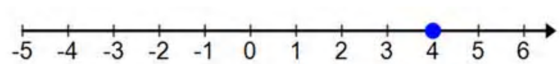
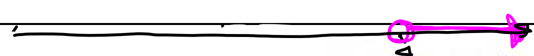
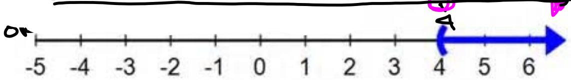
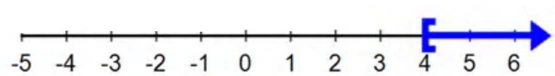
A compound inequality is a statement that involves more than one inequality.

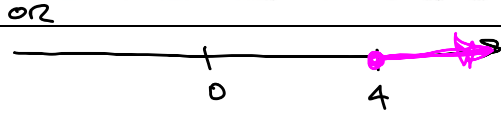
Examples of compound inequalities Note: $-3<x$ is equivalent to $x>-3$

$-3<x<5$ means $-3<x$ and $x<5$ means x is greater than -3 and x is less than 5.

$4<x\leq 9$ means $4<x$ and $x\leq 9$ means x is greater than 4 and x is less than or equal to 9.

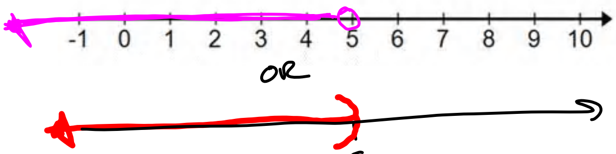
A number line is a useful tool to visualize the solution set of an equation or inequality.

Solution	Translation	Graph of Solution	Notes
$x=4$	x is equal to 4		Graph a number as a single point.
$x>4$	x is greater than 4	 or 	The parenthesis, (, is used on the graph to indicate that $x=4$ is *not* included.
$x\geq 4$	x is greater than or equal to 4	 or	The square bracket symbol, [, is used on the graph to indicate that $x=4$ is included.

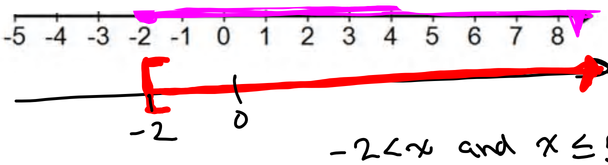


Graph the solution sets.

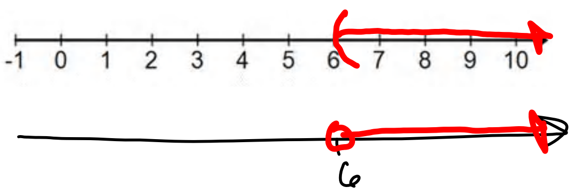
1. $x < 5$



2. $x \geq -2$



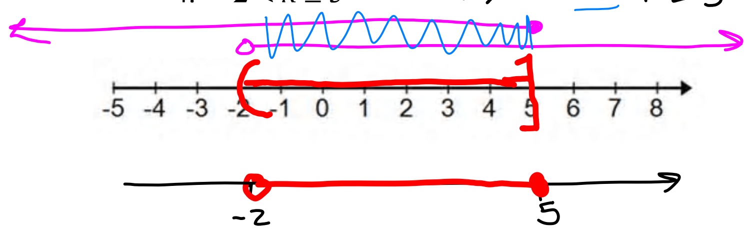
3. $6 < y$ Rewrite: $y > 6$



4. $-2 < x \leq 5$

$-2 < x$ and $x \leq 5$

$x > -2$ and $x \leq 5$



Set-Builder Notation and Interval Notation

Set-Builder Notation

Inequality: $x \geq 2$ Set-builder notation: $\{x \mid x \geq 2\}$

The set of all x such that x is greater than or equal to 2

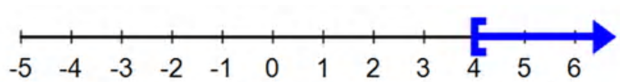
∞: infinity

Interval notation:

$[2, \infty)$

Interval Notation

Inequality: $x \geq 4$



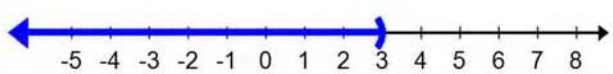
Interval Notation: $[4 \quad , \quad \infty)$

$[4, \infty)$

set builder notation:

$\{x \mid x \geq 4\}$

Inequality: $x < 3$

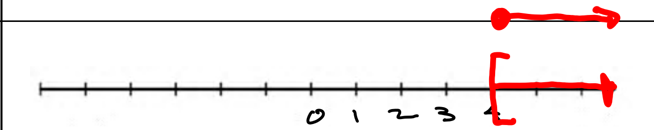
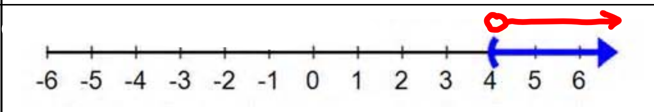
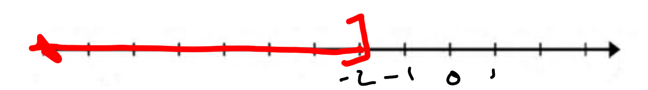
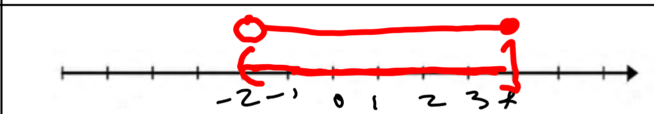
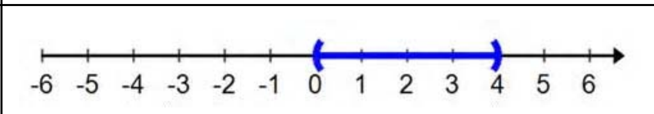


Interval Notation: $(-\infty \quad , \quad 3)$

$(-\infty, 3)$

Note: 2.99 is included in this set, but 3 is not.

5. Fill in the chart.

Set-Builder Notation	Graph	Interval Notation
5. $\{x \mid x \ge 4\}$		$[4, \infty)$
6. $\{x \mid x > 4\}$		$(4, \infty)$
7. $\{x \mid x \le -2\}$		$(-\infty, -2]$
8. $\{x \mid -2 < x \le 4\}$		$(-2, 4]$
9. $\{x \mid 0 < x < 4\}$		$(0, 4)$

Addition and Subtraction Properties of Inequality

Addition and Subtraction Properties of Inequality

Let *a*, *b*, and *c* represent real numbers.

If $a < b$, then $a + c < b + c$ you may add the same number on both sides

If $a < b$, then $a - c < b - c$ you may subtract the same number from both sides

These properties may also be stated for $a \le b$, $a > b$, and $a \ge b$.

---

10. Solve the inequality. Graph the solution set and write the set in interval notation.

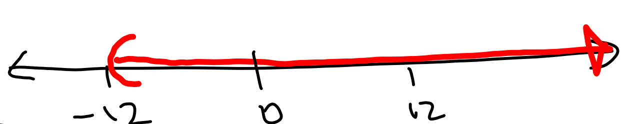
Ex: Solve: $5 < 7 + a$

$-7 \qquad -7$

$-12 < a$

Highly recommend: rewrite with the variable on the left:

$a > -12$



OR

Solution Set: $\{a \mid a > -12\}$

Interval Notation: $(-12, \infty)$

Multiplication and Division Properties of Inequality

Multiplication and Division Properties of Inequality

Let *a*, *b*, and *c* represent real numbers.

If *c* is positive and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$ — you may multiply or divide by the same ***positive*** number on both sides

If *c* is negative and $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$ — you may multiply or divide by the same ***negative*** number on both sides, but you MUST reverse the inequality sign

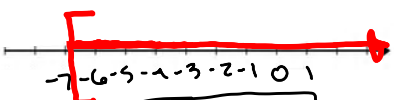
These properties may also be stated for $a \le b$, $a > b$, and $a \ge b$.

Solve the inequality. Graph the solution set and write the set in interval notation.

11. $-g \le 7$

$\frac{-g}{-1} \ge \frac{7}{-1}$ ← reverse the inequality sign!

$g \ge -7$



$[-7, \infty)$

$\{g \mid g \ge -7\}$

12. $8 < 2x - 10$

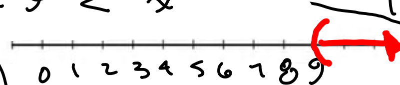
$+10 \quad +10$

$18 < 2x$

$\frac{18}{2} < \frac{2x}{2}$

$9 < x$

Rewrite as $x > 9$



$(9, \infty)$

$\{x \mid x > 9\}$

$3 > 5 - 2x$

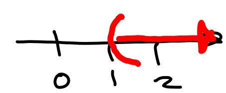
$-5 \quad -5$

$-2 > -2x$

$\frac{-2}{-2} < \frac{-2x}{-2}$

$1 < x$

Rewrite as $x > 1$



$\{x \mid x > 1\}$

$(1, \infty)$

13. $-1.1p - 0.3 < -0.3p + 2.1$

14. $\frac{2}{3}w + \frac{1}{6} \ge \frac{3}{4}w - \frac{5}{12}$

$\frac{2}{3}w + \frac{1}{6} \ge \frac{3}{4}w - \frac{5}{12}$

Multiply both sides by 12:

$\frac{12}{1}\left(\frac{2}{3}w\right) + \frac{12}{1}\left(\frac{1}{6}\right) \ge \frac{12}{1}\left(\frac{3}{4}w\right) - \frac{5}{12}\left(\frac{12}{1}\right)$

$8w + 2 \ge 9w - 5$

$-9w \quad -9w$

$-w + 2 \ge -5$

$-2 \quad -2$

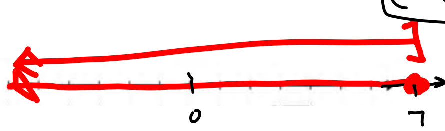
$-w \ge -7$

$\frac{-w}{-1} \le \frac{-7}{-1}$

$w \le 7$

$\{w \mid w \le 7\}$

Interval Notation: $(-\infty, 7]$



Why must we reverse the inequality sign when multiplying / dividing by a negative number?

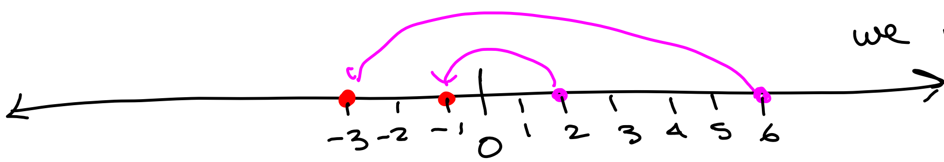
$2 < 6$ True

Divide by -2:

$$\frac{2}{-2} < \frac{6}{-2}$$

$-1 < -3$ False!

so the step above is false also (to avoid getting a false statement, we reverse the inequality sign)



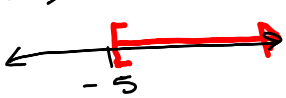
Ex: Solve.

$$4x \geq -20$$

$$\frac{4x}{4} \geq \frac{-20}{4}$$

$$x \geq -5$$

$\boxed{\{x \mid x \geq -5\}}$



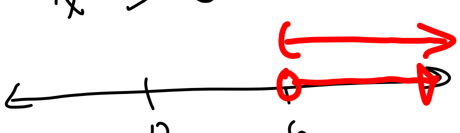
$\boxed{[-5, \infty)}$

Ex: Solve $-3x < -18$

$$\frac{-3x}{-3} > \frac{-18}{-3}$$ Reverse the inequality sign!

$$x > 6$$

$\boxed{\{x \mid x > 6\}}$

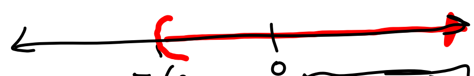


$\boxed{(6, \infty)}$

Ex: Solve. $-3x < 18$

$$\frac{-3x}{-3} > \frac{18}{-3}$$ Reverse the inequality!

$$x > -6$$



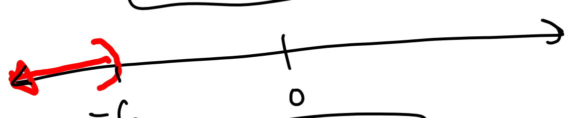
$\boxed{(-6, \infty)}$

Ex: $3x < -18$

$$\frac{3x}{3} < \frac{-18}{3}$$

$$x < -6$$

$\boxed{\{x \mid x < -6\}}$



$\boxed{(-\infty, -6)}$

15. Determine whether the given number is a solution to the inequality.

$3(x-1)+7>16+x; \quad x=6$

Inequalities of the Form $a<x<b$

(combined inequality)

To solve a compound inequality, isolate the variable x in the middle.

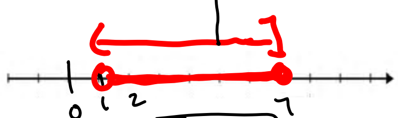
Note: The operations performed on the middle portion of the inequality must also be performed on the left-hand side and right-hand side.

Graph the solution.

16. $-2<x-3\le 4$

$+3 \quad +3 \quad +3$

$1<x\le 7$



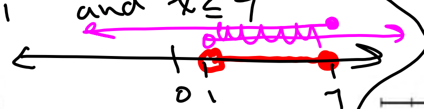
$\boxed{(1,7]}$

$-2<x-3$ and $x-3\le 4$

$+3 \quad +3 \qquad +3 \quad +3$

$1<x$ and $x\le 7$

$x>1$ and $x\le 7$



17. $-9<3y<12$

Applications of Linear Inequalities

(skip the applications)

Commonly used translations to express inequalities.

English Phrase	Mathematical Inequality
a is less than b	$a<b$
a is greater than b a exceeds b	$a>b$
a is less than or equal to b a is at most b a is no more than b	$a\le b$
a is greater than or equal to b a is at least b a is no less than b	$a\ge b$

For 18 - 21, translate the English phrases into mathematical inequalities.

18. The temperature in the classroom, t, was at most 75°F.

$t \le 75^\circ F$

19. The number of goals John scored, g, exceeded 4.

$g > 4$

20. Ann's weight, w, is between 120 lb and 130 lb.

$120 < w < 130$

22. A company sells boxes of chocolates for fundraising. The company sells the boxes for \$40 each. However, for large orders, the price per box is discounted by a percentage off the original price. Let x represent the number of boxes ordered. The corresponding discount is given in the table.

a. If a school orders 1000 boxes of chocolates, compute the total cost.

Number of Boxes Ordered	**Discount**
$x \le 500$	0%
$501 \le x \le 1000$	20%
$x \ge 1001$	25%

Skip these!

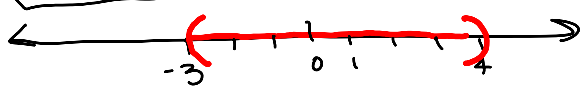
b. Which costs more: 500 boxes or 502 boxes? Explain your answer.

17. $-9<3y<12$

$\frac{-9}{3} < \frac{3y}{3} < \frac{12}{3}$

$-3 < y < 4$

$\boxed{\{y \mid -3<y<4\}}$



Interval notation: $\boxed{(-3,4)}$

Ex: $-1 < -4x-1 < 12$

Example: $4 \le 1-5x < 7$

$\quad -1 \quad -1 \quad -1$

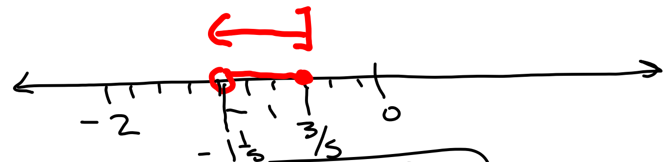
$3 \le -5x < 6$

Reverse the inequality signs $\longrightarrow$ $\frac{3}{-5} \ge \frac{-5x}{-5} > \frac{6}{-5}$

$-\frac{3}{5} \ge x > -1\frac{1}{5}$

Rewrite with smallest number on left:

$-1\frac{1}{5} < x \le -\frac{3}{5}$



$\boxed{(-1\frac{1}{5}, -\frac{3}{5}] \text{ or } (-\frac{6}{5}, -\frac{3}{5}]}$

$\boxed{\{x \mid -\frac{6}{5} < x \le -\frac{3}{5}\}}$

Ex: $-10 < 2x+4 < -14$

$\quad -4 \quad -4 \quad -4$

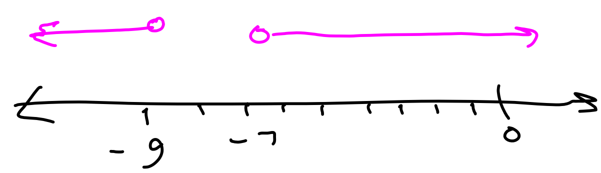
$-14 < 2x < -18$

$\frac{-14}{2} < \frac{2x}{2} < \frac{-18}{2}$

$-7 < x < -9$

$-7 < x$ and $x < -9$

$x > -7$

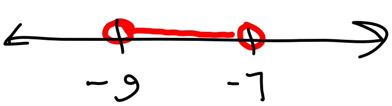


final answer

$\boxed{\text{No solution}}$

There is no x that satisfies both $x > -7$ and $x > -9$

Example:



write this in interval notation:

$\boxed{(-9,-7)}$

$\boxed{\{x \mid -9<x<-7\}}$

Ex: $-4 \le \frac{2}{3}x + \frac{1}{2} \le \frac{9}{2}$

multiply all 3 sides by -6:

$$-4(6) \le \frac{2}{3}x\,\frac{(6)}{1} + \frac{1}{2}\,\frac{(6)}{1} \le \frac{9}{2}\,\frac{(6)}{1}$$

$$-24 \le \frac{12}{3}x + \frac{6}{2} \le \frac{54}{2}$$

$$\begin{aligned} -24 &\le 4x + 3 \le 27 \\ {\scriptstyle -3} &\qquad\ \ {\scriptstyle -3} \qquad {\scriptstyle -3} \end{aligned}$$

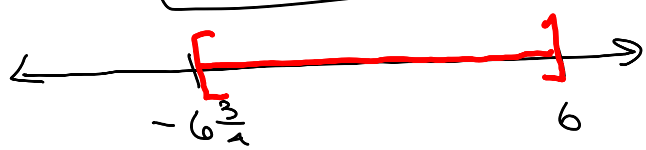
$$-27 \le 4x \le 24$$

$$\frac{-27}{4} \le \frac{4x}{4} \le \frac{24}{4}$$

$$-6\tfrac{3}{4} \le x \le 6$$

$$\boxed{\{x \mid -6\tfrac{3}{4} \le x \le 6\}}$$

$$\boxed{\left[-6\tfrac{3}{4},\ 6\right]}$$



Homework Qs

2.8 #97 Determine whether the given number is a solution to the inequality.

$$-2x + 5 < 4; \quad x = -2$$

we need to decide if -2 makes the inequality true.

$$-2x + 5 < 4$$

substitute $x = -2$: $-2(-2) + 5 < 4$

$$4 + 5 < 4$$

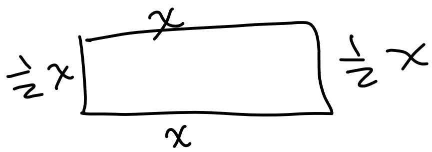
$$9 < 4 \quad \text{False}$$

$\boxed{\text{No, it is not a solution.}}$

2.6 #44 The width of a workbench is $\frac{1}{2}$ the length. The perimeter is 240 in. Find the length, width.

length: x
width: $\frac{1}{2}x$

width $\xrightarrow[\text{to}]{\text{compared}}$ length
x



$$\frac{1}{2}x + x + \frac{1}{2}x + x = 240$$

$$x + x + x = 240$$

$$3x = 240$$

1.6 #109 Simplify.

$$-\frac{1}{3}(6t+9)+10$$

$$= -\frac{1}{3}(6t) - \frac{1}{3}(9) + 10$$

$$= -\frac{6}{3}t - \frac{9}{3} + 10$$

$$= -2t - 3 + 10$$

$$= \boxed{-2t+7}$$

1.6 #111 Simplify.

$$10(5.1a - 3.1) + 4$$

$$= 51a - 31 + 4$$

$$= \boxed{51a - 27}$$

Ex:

$$3.5(2x+1.5) - 7.5$$