Definition of a Rational Expression

Rational Number: The ratio of two integers, $\frac{p}{q}$, where $q \neq 0$.

Rational Numbers: $\frac{1}{3}, -\frac{3}{4}, \frac{7}{5}, 4 \left(\frac{4}{1}\right), 0.35 \left(\frac{35}{100}\right), 0.\overline{3} \left(\frac{1}{3}\right)$

Rational Expression: The ratio of two polynomials, $\frac{p}{q}$, where $q \neq 0$.

Irrational numbers (can't be written as a ratio of 2 integers): $\sqrt{65}, \sqrt{2}, \pi, e$

Examples of rational expressions: $\frac{2x+5}{x^2+x-1}$, $\frac{4}{5}$, $\frac{5t^3+4t+2}{3t^2}$, $\frac{7}{t+1}$

Evaluating Rational Expressions

For exercises 1 and 2, evaluate the rational expression (if possible) for the given values.

1. $\frac{r+2}{r^2+2r+5}$; $r=3$

$r=3 \Rightarrow \frac{3+2}{3^2+2(3)+5} = \frac{5}{9+6+5} = \frac{5}{20} = \boxed{\frac{1}{4}}$

2. $\frac{(w-2)(w-3)}{(w+1)(w+4)}$; $w=-4$

$w=-4 \Rightarrow \frac{(-4-2)(-4-3)}{(-4+1)(-4+4)} \Rightarrow \frac{(-6)(-7)}{(-3)(0)} \Rightarrow \frac{42}{0}$ undefined

3. Jan's cake company has a fixed monthly cost of \$3500, plus a variable cost of \$8 per cake. The average cost per cake, C (in dollars), is given by the equation $C = \frac{3500+8x}{x}$ where x represents the number of cakes produced.

a. Find the average cost per cake if Jan's company produces 500 cakes.

skip?

b. Find the average cost per cake if Jan's company produces 750 cakes.

c. Find the average cost per cake if Jan's company produces 1000 cakes.

Restricted Values of a Rational Expression

Restricted values of a rational expression: The values that make the expression undefined, that is, make the denominator equation to zero.

For exercises 4 – 7, identify the restricted values. (Set the denominator equal to zero and solve the resulting equation.)

4. $\frac{7}{g-2}$

$g \neq 2$

$g=0$ is OK:

$\frac{7}{0-2} = \frac{7}{-2} = -\frac{7}{2}$

5. $\frac{f-5}{f^2-2f-8} = \frac{f-5}{(f-4)(f+2)}$

$f \neq 4, f \neq -2$

6. $\frac{4x}{x^2+16}$

No restrictions

Note: x^2+16 is always positive

7. $\frac{2a-5}{a}$

$a \neq 0$

8. Construct a rational expression that is undefined for $x=-3$ and $x=1$.

$\frac{12x^3+72x+11}{(x+3)(x-1)}$

Simplifying Rational Expressions

To simplify a rational expression to lowest term, we simplify the ratio of common factors to 1.

Fundamental Principle of Rational Expressions

Let p, q, and r represent polynomials where $q \neq 0$ and $r \neq 0$. Then

$$\frac{pr}{qr} = \frac{p}{q} \cdot \frac{r}{r} = \frac{p}{q} \cdot 1 = \frac{p}{q}$$

Note: The restricted values are always determined before the expression is simplified.

Note:

$\frac{16}{24} = \frac{2 \cdot 8}{3 \cdot 8} = \frac{2}{3} \cdot \frac{8}{8} = \frac{2}{3}$

For exercises 9 – 10, a. Identify the restricted values. b. Simplify the expression to lowest terms.

9. $\frac{6x-12}{9x-18}$

a) Restricted values $x-2\neq 0$

$x \neq 2$

$\frac{6(x-2)}{9(x-2)} = \frac{6}{9} = \frac{2}{3}$

b) $\frac{2}{3}$

10. $\frac{15x-20}{9x^2-16}$

$\frac{5(3x-4)}{(3x-4)(3x+4)}$

$= \frac{5}{3x+4}$

$9x^2-16$

$(3x)^2-(4)^2$

$(3x-4)(3x+4)$

$[a^2-b^2 = (a+b)(a-b)]$

a) Find restricted values for #10

$3x-4\neq 0$ $\quad 3x+4\neq 0$

$3x\neq 4$ $\quad 3x\neq -4$

$x\neq \frac{4}{3}$ $\quad \frac{3x}{3}\neq \frac{-4}{3}$

$x\neq -4/3$

Restricted values: $x\neq \frac{4}{3}, x\neq -\frac{4}{3}$

For exercises 11 – 16, simplify the expression to lowest terms.

11. $\frac{30a^3b^4}{36a^5b}$

$= \frac{5b^3}{6a^2}$

12. $\frac{6c(c+2)(c-1)}{10c^3(c-1)^2(c+2)}$

$= \frac{3c(c+2)(c-1)}{5c^3(c-1)(c-1)(c+2)} = \frac{3}{5c^2(c-1)}$

13. $\frac{12x^2-20x}{24x^2y+40xy}$

$= \frac{4x(3x-5)}{8xy(3x+5)}$

$= \frac{1(3x-5)}{2y(3x+5)} = \frac{3x-5}{2y(3x+5)}$

14. $\frac{7x+14}{x^2-4x-12}$

$= \frac{7(x+2)}{(x+2)(x-6)} = \frac{7}{x-6}$

15. $\frac{h+2}{h^2+4}$

Does not simplify.

Simplified version is $\frac{h+2}{h^2+4}$

Recall: h^2+ positive is prime.

16. $\frac{2x^2+x-6}{ax^2-bx^2-4a+4b}$

$= \frac{(2x-3)(x+2)}{(a-b)(x+2)(x-2)}$

$= \frac{2x-3}{(a-b)(x-2)}$

Scratchwork

Denominator:

$ax^2-bx^2-4a+4b$

$(ax^2-bx^2)+(-4a+4b)$

$x^2(a-b)-4(a-b)$

$(a-b)(x^2-4)$

$(a-b)(x+2)(x-2)$

#16 numerator: $2x^2+x-6$

$2x^2$ $\quad 6$

$2x\cdot x$ $\quad 1\cdot 6$, $2\cdot 3$

$4x$ $\quad 3x$

$(2x-3)(x+2)$

Check: $2x^2+4x-3x-6$

$2x^2+x-6$ ✓

Simplifying a Ratio of –1

When two factors are opposites, they form a ratio of -1.

17. Identify which pairs are opposites.

a. $a-3$ and $3-a$ b. $w+6$ and $6+w$ c. $x-5$ and $-5+x$

opposites

$a-3 = -1(3-a)$

For exercises 18 – 21, simplify to lowest terms.

18. $\dfrac{5+y}{-5-y} = \dfrac{5+y}{-1(5+y)}$

$\dfrac{1}{-1(1)} = \dfrac{1}{-1} = \boxed{-1}$

19. $\dfrac{b+6}{b-6}$

nothing I can do.

$\boxed{\dfrac{b+6}{b-6}}$

20. $\dfrac{64-g^2}{g^2-5g-24} = \dfrac{-1(g^2-64)}{g^2-5g-24}$

$= \dfrac{-1(g+8)(g-8)}{(g-8)(g+3)} = \dfrac{-(g+8)}{g+3}$

$= \boxed{-\dfrac{g+8}{g+3}}$

Recall: $a-b = -1(b-a)$

check: $-1(b-a) = -b+a = a-b$

21. $\dfrac{4x-8}{12-6x}$

$\dfrac{4(x-2)}{6(2-x)}$

$= \dfrac{4(x-2)}{6(-1)(-2+x)}$

$= \dfrac{4(x-2)}{-6(x-2)}$

$= -\dfrac{4}{6} = \boxed{-\dfrac{2}{3}}$

Question from Review Sheet

\# 25) $36x^4 - 18x^3 + 45x^2 + 27x$

either find GCF 1st or group 1st.

Group first:

$(36x^4 - 18x^3) + (45x^2 + 27x)$

$9x^3(4x - 2) + 9x(5x + 3)$

$9x^3(2)(2x - 1) + 9x(5x + 3)$ doesn't work

Change the order:

$36x^4 + 45x^2 - 18x^3 + 27x$

$9x^2(4x^2 + 5) - 9x(2x^2 - 3)$

parentheses still don't match

So this polynomial doesn't factor by grouping

So, pulling out GCF is only option:

$36x^4 - 18x^3 + 45x^2 + 27x$

$\boxed{9x(4x^3 - 2x^2 + 5x + 3)}$

Ex: $-16x^2y - 4x^2 + 24yx + 6x$

Here, you could pull out GCF 1st or group 1st:

Grouping 1st:

$(-16x^2y - 4x^2) + (24yx + 6x)$

$-4x^2(4y + 1) + 6x(4y + 1)$

$(4y+1)(-4x^2 + 6x)$

$(4y+1)(-2x)(2x - 3)$

$\boxed{-2x(4y+1)(2x-3)}$

OR, take out GCF 1st:

$-(16x^2y - 4x^2 + 24yx + 6x$

$-2x(8xy + 2x - 12y - 3)$

$-2x[(8xy + 2x) + (-12y - 3)]$

$-2x[2x(4y+1) - 3(4y+1)]$

$-2x[(4y+1)(2x-3)]$

$\boxed{-2x(4y+1)(2x-3)}$