Multiplication of Rational Expressions

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Let p, q, r and s represent polynomials, such that $q \neq 0$, $s \neq 0$. Then $\frac{p}{q} \cdot \frac{r}{s} = \frac{pr}{qs}$

Ex: $\frac{16}{15} \cdot \frac{21}{32} = \frac{7}{10}$

Multiplying Rational Expressions

Step 1 Factor the numerators and denominators of all rational expressions.

Step 2 Simplify the ratios of common factors to 1.

Step 3 Multiply the remaining factors in the numerator, and multiply the remaining factors in the denominator.

For exercises 1 – 4, multiply.

1. $\frac{8x^2y}{30x^3y^5} \cdot \frac{5y^2}{7x^3}$

$\frac{8x^2y}{30x^3y^5} \cdot \frac{5y^2}{7x^3} = \frac{8x^2y^3}{42x^6y^5} = \frac{4}{21x^4y^2}$

2. $\frac{4a+8}{a^3} \cdot \frac{3a}{6a+12}$

$\frac{4(a+2)}{a^3} \cdot \frac{3a}{6(a+2)} = \frac{4a}{2a^3} = \frac{2}{a^2}$

3. $\frac{9-4a^2}{4a-6} \cdot \frac{4a}{2a^2+5a+3}$

$\frac{-4a^2+9}{2(2a-3)} \cdot \frac{4a}{(2a+3)(a+1)}$

$\frac{-1(4a^2-9)}{2(2a-3)} \cdot \frac{4a}{(2a+3)(a+1)}$

$\frac{-1(2a-3)(2a+3)}{2(2a-3)} \cdot \frac{4a}{(2a+3)(a+1)} = \frac{-2a}{a+1}$

4. $\frac{7b^2-7}{b^2-9} \cdot \frac{(b+3)^2}{14b^2-14b}$

Division of Rational Expressions

Division of Rational Expressions

Let p, q, r and s represent polynomials, such that $q \neq 0$, $r \neq 0$, $s \neq 0$. Then $\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \cdot \frac{s}{r} = \frac{ps}{qr}$

Dividing by a number is the same as multiplying by its reciprocal.

For exercises 5 – 8, divide.

5. $\frac{4a^3}{10a^2b} \div \frac{a^2b^2}{5b^5}$

$\frac{4a^3}{10a^2b} \cdot \frac{5b^5}{a^2b^2}$

$= \frac{20a^3b^5}{10a^4b^3}$

$= \frac{2b^2}{1a} = \boxed{\frac{2b^2}{a}}$

6. $\frac{\frac{14c^3d^4}{5c^2}}{\frac{7d^3}{20c^2d}}$

$= \frac{14c^3d^4}{5c^2} \div \frac{7d^3}{20c^2d}$

$= \frac{14c^3d^4}{5c^2} \cdot \frac{20c^2d}{7d^3}$

$= \frac{8c^5d^5}{c^2d^3} = \frac{8c^3d^2}{1} = \boxed{8c^3d^2}$

7. $\frac{12x-24}{x^2-4x+4} \div \frac{4x}{x+2}$

$\frac{12x-24}{x^2-4x+4} \cdot \frac{x+2}{4x}$

$\frac{12(x-2)}{(x-2)(x-2)} \cdot \frac{x+2}{4x}$

$\boxed{\frac{3(x+2)}{x(x-2)}} = \boxed{\frac{3x+6}{x^2-2x}}$

both are correct

8. $\frac{3m^2-27}{m^2-2m-3} \div \frac{6m+18}{m^2+2m+1}$

For exercises 9 – 14, multiply or divide as indicated.

9. $\dfrac{2m+1}{2m^2+11m-6}\cdot(m+6)$

$\dfrac{2m+1}{2m^2+11m-6}\cdot\dfrac{m+6}{1}$

10. $\dfrac{4y^2-y-3}{3y-3}\div(4y+3)$

$\dfrac{4y^2-y-3}{3y-3}\div\dfrac{4y+3}{1}$

$\dfrac{4y^2-y-3}{3y-3}\cdot\dfrac{1}{4y+3}$

11. $\dfrac{a^2+3a-18}{a^2+13a+40}\div\dfrac{a-3}{a+5}$

12. $\dfrac{ax-a+2bx-2b}{a^2-4b^2}\cdot\dfrac{b^2-4ab+4a^2}{3x^2-3x}$

13. $\dfrac{b^3+2b^2-b-2}{b^2+4b+3} \div \dfrac{b^2-4}{2b^2+5b-3} \cdot \dfrac{3b+6}{4b^2-1}$

14. $\dfrac{p^2+3p}{p^2+7p+12} \div \dfrac{p^2+p}{p^2-16} \div \dfrac{4p+4}{10p+40}$