Addition and Subtraction of Rational Expressions with the Same Denominator

Note: To add or subtract rational expressions, the expressions must have the same denominator.

PROPERTY Addition and Subtraction of Rational Expressions

Let p, q, and r represent polynomials where $q \neq 0$. Then,

1. $\frac{p}{q}+\frac{r}{q}=\frac{p+r}{q}$ 2. $\frac{p}{q}-\frac{r}{q}=\frac{p-r}{q}$

Ex: $\frac{2}{7}+\frac{3}{7}=\frac{2+3}{7}=\boxed{\frac{5}{7}}$

Note: To add or subtract rational expressions with the same denominator:

- combine the terms in the numerator
- write the result over the common denominator
- if possible, simplify the expression to lowest terms

For exercises 1 – 6, add or subtract the expressions with like denominators as indicated.

1. $\frac{7}{10}-\frac{2}{10}=\frac{7-2}{10}=\frac{5}{10}=\boxed{\frac{1}{2}}$

Ex: $\frac{3}{8}-\frac{2}{8}=\frac{3-2}{8}=\boxed{\frac{1}{8}}$

2. $\frac{3a}{a-4}-\frac{a+8}{a-4}=\frac{(3a)}{a-4}-\frac{(a+8)}{a-4}$

$\frac{3a-(a+8)}{a-4}=\frac{3a-a-8}{a-4}$

$=\frac{2a-8}{a-4}=\frac{2\cancel{(a-4)}}{\cancel{a-4}}$

$=\frac{2}{1}=\boxed{2}$

3. $\frac{4c}{c+5}+\frac{20}{c+5}$

$=\frac{4c+20}{c+5}=\frac{4\cancel{(c+5)}}{\cancel{c+5}}$

$=\frac{4}{1}=\boxed{4}$

4. $\frac{d^2}{d-1}-\frac{8d-7}{d-1}$

$\frac{d^2}{d-1}-\frac{(8d-7)}{d-1}=\frac{d^2-(8d-7)}{d-1}$

$=\frac{d^2-8d+7}{d-1}=\frac{\cancel{(d-1)}(d-7)}{\cancel{d-1}}$

$=\frac{d-7}{1}=\boxed{d-7}$

5. $\dfrac{c^2}{c-6} - \dfrac{36}{c-6}$

$$\frac{c^2-36}{c-6} = \frac{(c+6)\cancel{(c-6)}}{\cancel{c-6}}$$

$$= \frac{c+6}{1} = \boxed{c+6}$$

6. $\dfrac{4}{3x^2+2x-8} + \dfrac{-3x}{3x^2+2x-8}$

$$\frac{4-3x}{3x^2+2x-8} = \frac{4-3x}{(3x-4)(x+2)}$$

$$= \frac{-3x+4}{(3x-4)(x+2)} = \frac{-1\cancel{(3x-4)}}{\cancel{(3x-4)}(x+2)}$$

$$\boxed{\frac{-1}{x+2}} = \boxed{-\frac{1}{x+2}}$$

Addition and Subtraction of Rational Expressions with Different Denominators

Note: To add or subtract two rational expressions, the expressions must have the same denominator.

PROCEDURE Adding or Subtracting Rational Expressions

Step 1 Factor the denominators of each rational expression.

Step 2 Identify the LCD

Ex.: $\dfrac{2}{7} + \dfrac{1}{3} = \dfrac{2}{7}\left(\dfrac{3}{3}\right) + \dfrac{1}{3}\left(\dfrac{7}{7}\right) = \dfrac{6}{21} + \dfrac{7}{21} = \boxed{\dfrac{13}{21}}$

Step 3 Rewrite each rational expression as an equivalent expression with the LCD as its denominator

Step 4 Add or subtract the numerators, and write the result over the common denominator.

Step 5 Simplify to lowest terms.

For exercises 7 – 13, add or subtract the expressions with unlike denominators as indicated.

7. $\dfrac{4}{a^2b^4} + \dfrac{2}{a^4b^3}$

LCD: a^4b^4

$$\frac{4}{a^2b^4} + \frac{2}{a^4b^3}$$

$$\frac{4}{a^2b^4}\left(\frac{a^2}{a^2}\right) + \frac{2}{a^4b^3}\left(\frac{b}{b}\right)$$

$$= \boxed{\frac{4a^2+2b}{a^4b^4}} = \boxed{\frac{2(2a^2+b)}{a^4b^4}}$$

8. $\dfrac{4}{5t+10} + \dfrac{6}{t+2}$

LCD: $5(t+2)$

$$= \frac{4}{5(t+2)} + \frac{6}{t+2}$$

$$= \frac{4}{5(t+2)} + \frac{6}{t+2}\left(\frac{5}{5}\right)$$

$$= \frac{4}{5(t+2)} + \frac{30}{5(t+2)}$$

$$= \frac{4+30}{5(t+2)} = \boxed{\frac{34}{5(t+2)}}$$

LCD: $y(y-8)$

9. $\dfrac{y}{y-8}+\dfrac{4}{y}$

$$\frac{y}{y-8}\left(\frac{y}{y}\right)+\frac{4}{y}\left(\frac{y-8}{y-8}\right)$$

$$\frac{y^2}{y(y-8)}+\frac{4y-32}{y(y-8)}$$

$$=\frac{y^2+(4y-32)}{y(y-8)}=\frac{y^2+4y-32}{y(y-8)}$$

$$=\boxed{\frac{(y+8)(y-4)}{y(y-8)}}$$

LCD: $2m(m-4)$

10. $\dfrac{24}{m^2-4m}-\dfrac{3m}{2m-8}$

$$\frac{24}{m(m-4)}+\frac{-3m}{2(m-4)}$$

$$\frac{24}{m(m-4)}\left(\frac{2}{2}\right)+\frac{-3m}{2(m-4)}\left(\frac{m}{m}\right)$$

$$\frac{48}{2m(m-4)}+\frac{-3m^2}{2m(m-4)}$$

$$\frac{48-3m^2}{2m(m-4)}=\frac{-3m^2+48}{2m(m-4)}=\frac{-3(m^2-16)}{2m(m-4)}$$

$$=\frac{-3(m+4)\cancel{(m-4)}}{2m\cancel{(m-4)}}$$

$$=\boxed{\frac{-3(m+4)}{2m}}$$

11. $\dfrac{3}{x^2+5x+6}+\dfrac{3}{x^2+7x+12}$

LCD: $(x+2)(x+3)(x+4)$

$$\frac{3}{(x+2)(x+3)}+\frac{3}{(x+3)(x+4)}$$

$$\frac{3}{(x+2)(x+3)}\left(\frac{x+4}{x+4}\right)+\frac{3}{(x+3)(x+4)}\left(\frac{x+2}{x+2}\right)$$

$$\frac{3x+12+3x+6}{(x+2)(x+3)(x+4)}=\frac{6x+18}{(x+2)(x+3)(x+4)}$$

$$=\frac{6\cancel{(x+3)}}{(x+2)\cancel{(x+3)}(x+4)}=\boxed{\frac{6}{(x+2)(x+4)}}$$

12. $\dfrac{p-3}{p^2+3p+2}+\dfrac{p-1}{p^2-4}$

LCD: $(p+2)(p+1)(p-2)$

$$\frac{p-3}{(p+2)(p+1)}+\frac{p-1}{(p+2)(p-2)}$$

$$\frac{p-3}{(p+2)(p+1)}\left(\frac{p-2}{p-2}\right)+\frac{p-1}{(p+2)(p-2)}\left(\frac{p+1}{p+1}\right)$$

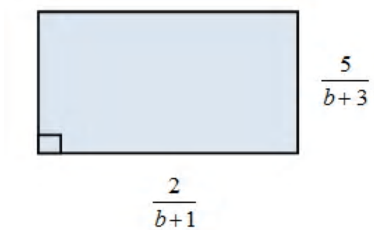
$$\frac{p^2-2p-3p+6+p^2+1p-1p-1}{(p+2)(p+1)(p-2)}$$

$$=\frac{p^2-5p+6+p^2-1}{(p+2)(p+1)(p-2)}$$

$$=\boxed{\frac{2p^2-5p+5}{(p+2)(p+1)(p-2)}}$$

13. $\dfrac{2}{c+2}-\dfrac{3}{c}+\dfrac{c+10}{c^2-4}$

14. Find an expression that represents the perimeter of the figure. Assume that $b > 0$.



Skip the applications

Using Rational Expressions in Translations

15. Write the reciprocal of the difference of a number and 4.

16. Write the quotient of 6 and the sum of 3 and a number.

For exercises 17 and 18, translate the English phrases into algebraic expressions. Then simplify by combining the rational expressions.

17. The sum of the reciprocal of a number and the quotient of 5 and twice the number.

18. The difference of a number and six times the reciprocal of the number.