

## Commutative Properties of Real Numbers

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Commutative Property of Addition

$$a + b = b + a$$

(The order in which two numbers are added does not affect the sum.)

Commutative Property of Multiplication

$$ab = ba$$

(The order in which two numbers are multiplied does not affect the product.)

Note: The operations of subtraction and division are not commutative.

$$\rightarrow 5 - 3 \neq 3 - 5 \quad \left| \quad \frac{6}{2} \neq \frac{2}{6} \right.$$

Use the commutative property of addition or multiplication to rewrite each expression.

1.  $-m(x)$   
 $-m(x)$   
 $= -xm$

2.  $a - 17$   
 $= -17 + a$

3.  $b(-52)$   
 $= -52b$

4.  $5a + 12z$   
 $= 12z + 5a$

## Associative Properties of Real Numbers

### Commutative Properties of Real Numbers

Associative Property of Addition

$$(a + b) + c = a + (b + c)$$

(The manner in which three numbers are grouped under addition does not affect the sum.)

Associative Property of Multiplication

$$(ab)c = a(bc)$$

(The manner in which three numbers are grouped under multiplication does not affect the sum.)

Use the associative property of addition or multiplication to rewrite each expression. Then simplify the expression.

5.  $-4(13a)$   
 $= (-4 \cdot 13)a$   
 $= -52a$

6.  $(4b + 13) + (-8)$   
 $= 4b + 13 - 8$   
 $= \boxed{4b + 5}$

7.  $-27(-2x)$   
 $= (-27)(-2)x$   
 $= \boxed{54x}$

Distributive Prop.  $-4(13 + a) = \boxed{-52 - 4a}$

## Identity and Inverse Properties of Real Numbers

### Identity and Inverse Properties of Real Numbers

If  $a$  is a real number and  $b$  is a nonzero real number, then

Identity Property of Addition

$$a + 0 = 0 + a = a$$

Identity Property of Multiplication

$$a \cdot 1 = 1 \cdot a = a$$

Inverse Property of Addition

$$a + (-a) = -a + a = 0$$

Inverse Property of Multiplication

$$b \cdot \frac{1}{b} = \frac{1}{b} \cdot b = 1$$

additive identity  
 $\Rightarrow -a$  is the additive inverse of  $a$   
 $\frac{1}{b}$  is the multiplicative inverse of  $b$   
 multiplicative identity

0 is said to be the *additive identity* and 1 is said to be the *multiplicative identity*.

## Distributive Property of Multiplication over Addition

### Distributive Property of Multiplication over Addition

If  $a$ ,  $b$ , and  $c$  are real numbers, then

$$a(b+c) = ab+ac \quad \text{and} \quad (b+c)a = ab+ac$$

Use the distributive property to rewrite each expression.

$$8. \quad 5(a-3) = \boxed{5a - 15}$$

$$9. \quad 6(-9x+8y-12z) \\ = 6(-9x) + 6(8y) + 6(-12z) \\ = \boxed{-54x + 48y - 72z}$$

$$10. \quad -(-6x+3y) \\ = -1(-6x+3y) \\ = -1(-6x) - 1(3y) \\ = \boxed{6x - 3y}$$

why is  $\frac{3x}{2} = \frac{3}{2}x$ ?

$$\frac{3x}{2} = \frac{3}{2} \cdot \frac{x}{1} \\ = \frac{3x}{2}$$

$$11. \quad -\frac{3}{4}(2x-16) \\ = -\frac{3}{4}(2x) - \frac{3}{4}(-16) \\ = -\frac{6x}{4} + \frac{48}{4} = -\frac{3x}{2} + 12 \\ = \boxed{-\frac{3x}{2} + 12}$$

## Algebraic Expressions

### Term

A *term* is a constant or the product or quotient of constants and variables.

Example:  $5x^3$  is a *variable term*. 5 is the numerical *coefficient* of the term  $5x^3$ .  
12 is a *constant term*.

### Algebraic Expression

An *algebraic expression* is the sum of one or more terms.

### Like Terms

Terms are *like terms* if they have the same variables and the corresponding variables are raised to the same powers.

Which of the pairs are *like terms*?

Terms: expressions being added  
Factors: expressions being multiplied

$$12. \quad 4b, \frac{2}{3}b$$

Like terms

$$13. \quad 8xy^2, 2x^2y$$

Not like terms

$$14. \quad 3y, 3$$

Not like terms

Two terms can be added or subtracted only if they are *like terms*.

Example:  $5y+6y=(5+6)y=11y$   
using the distributive property

$7z-10z=-3z$   
in one step

We can only combine like terms. We combine terms by adding the coefficients.

Ex:

$$5 - 3 = \boxed{2}$$

$$3 - 5 = \boxed{-2}$$

$$-3 - 5 = \boxed{-8}$$

$$3 + 5 = \boxed{8}$$

$$-3 + 5 = \boxed{2}$$

same as

$$5 - 3 = \boxed{2}$$

$$5(-3) = \boxed{-15}$$

$$3(-5) = \boxed{-15}$$

$$-3(-5) = \boxed{15}$$

$$3(5) = \boxed{15}$$

$$\underline{\text{Ex:}} \quad -0.6x + 1.3x \\ = \boxed{0.7x}$$

Simplify by combining like terms.

$$15. \quad 6x + 14x \\ = \boxed{20x}$$

$$16. \quad \underline{-0.3a} + 2b - \underline{1.5a} \\ = -0.3a - 1.5a + 2b \\ = \boxed{-1.8a + 2b}$$

$$17. \quad 12 - 6x^2 - 8 + x^2 \\ = -6x^2 + x^2 + 12 - 8 \\ = -6x^2 + 1x^2 + 12 - 8 \\ = \boxed{-5x^2 + 4}$$

$$18. \quad \frac{1}{3}x - 2y + \frac{2}{3}x + \frac{1}{4}y \\ \frac{1}{3}x + \frac{2}{3}x - 2y + \frac{1}{4}y \\ \frac{3}{3}x - \frac{2}{1}y \left(\frac{1}{4}\right) + \frac{1}{4}y \\ = 1x - \frac{8}{4}y + \frac{1}{4}y = \boxed{x - \frac{7}{4}y}$$

Same as  $x - 1.75y$   
(because  $\frac{7}{4} = 1\frac{3}{4}$ )

Clear parentheses and combine like terms.

$$19. \quad 3(x+8) - 43 \\ = 3x + 24 - 43 \\ = \boxed{3x - 19}$$

$$\frac{43}{-24} \\ 19$$

$$20. \quad -23 + 12(-3x+5) \\ = -23 - 36x + 60 \\ = \boxed{-36x + 37}$$

$$21. \quad -16 - 9(21 - 2n) \\ = -16 - 9(21 - 2n) \\ = -16 - 189 + 18n \\ = \boxed{-205 + 18n}$$

$$22. \quad 3(-5y+2) + 7(y-4) \\ = -15y + 6 + 7y - 28$$

$$23. \quad \frac{1}{8}(16x+24) - \frac{3}{4}(2x+12) \\ \frac{1}{8}(16x) + \frac{1}{8}(24) - \frac{3}{4}(2x) - \frac{3}{4}(12) \\ = \frac{16}{8}x + \frac{24}{8} - \frac{6}{4}x - \frac{36}{4} \\ = 2x + 3 - \frac{3}{2}x - 9 \\ = \frac{2x}{1} \left(\frac{1}{2}\right) - \frac{3}{2}x + 3 - 9 \\ = \frac{4}{2}x - \frac{3}{2}x - 6 = \boxed{\frac{1}{2}x - 6}$$

$$24. \quad 5x - 3[x + 2(3x-6)] \\ = 5x - 3[x + 6x - 12] \\ = 5x - 3(7x - 12) \\ = 5x - 21x + 36 \\ = \boxed{-16x + 36}$$

Caution: Never write these:  
 $\frac{3}{4}x$  or  $3/4x$  ~~or~~

Does this mean  $\frac{3}{4}x = \frac{3x}{4}$   
or  $\frac{3}{4x}$

$\frac{3}{4}x = \frac{3}{4} \cdot \frac{x}{1} = \frac{3x}{4}$  - The  $x$  in  $\frac{3}{4}x$  is  
really in the numerator.

It's best to avoid diagonal fraction bars.